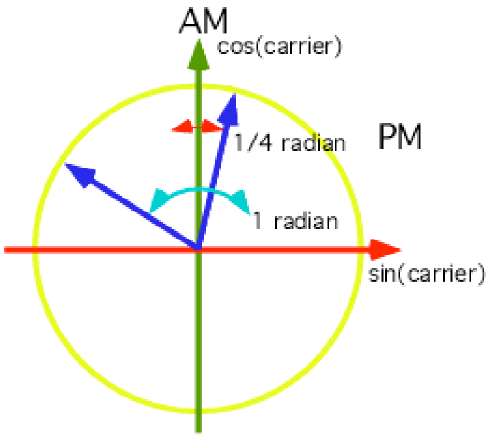
FULLY VISUALIZE A FM SPECTRUM



What if PM is generated by four Quad modulating a carrier to produce the right magnitude and phase? Start off doing a polar to rectangular conversion of a unity magnitude vector which has its angle being modulated by a 0.25radian sine wave. The Y direction represents the signal to be modulated by the cosine of the carrier. The X direction represents how the sinewave carrier will be modulated. The following Macspice simulation will do just that.

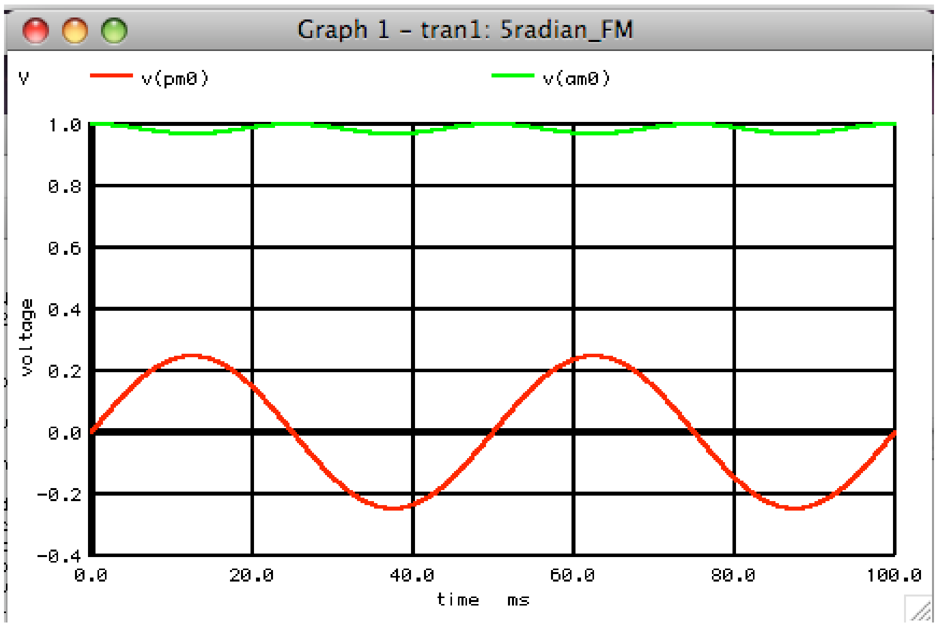
================MacSpice==========================

5radian_FM

```
V_SIN0 VSIN0 0 SIN(0 .25 20 0 )
BSIN0   PM0   0 v = sin(v(VSIN0))
BCOS0   AM0   0 v = cos(v(VSIN0))
V_SIN1 VSIN1 0 SIN(0 1 20 0 )
BSIN1   PM1   0 v = sin(v(VSIN1))
BCOS1   AM1   0 v = cos(v(VSIN1))
V_SIN2 VSIN2 0 SIN(0 2 20 0 )
BSIN2   PM2   0 v = sin(v(VSIN2))
BCOS2   AM2   0 v = cos(v(VSIN2))
V_SIN5 VSIN5 0 SIN(0 5 20 0 )
BSIN5   PM5   0 v = sin(v(VSIN5))
BCOS5   AM5   0 v = cos(v(VSIN5))
.tran 10u .1 0 10u
.control
set    pensize = 2
run
plot   v(pm0)  v(am0)
```

```
plot    v(pm1)  v(am1)
plot    v(pm2)  v(am2)
plot    v(pm5)  v(am5)
print mean(am0) mean(pm0)
linearize
set       specwindow=   "none"
spec      10          1k      10     v(am0)
plot      mag(v(am0))     loglog  title AM5
.endc
.end
======================================================================
```

In the case of +/-0.25radians phase modulation, the magnitude of the carrier in the cosine direction remains pretty much around one. In the sine direction the magnitude comes close to the magnitude of the phase modulation.



If the am0 and pm0 waveforms get put through a FFT, the harmonics magnitudes can all be printed out.

```
mean(am0) = 9.844484e-01
mean(pm0) = 3.107690e-08
```

MacSpice 2 -> print am0

```
--------------------------------------------------------------------------------
Index   am0                                 Numb Harmonic
--------------------------------------------------------------------------------
3       1.554379e-02,   2.749499e-11    4    2
7       2.028169e-05,   4.785207e-13    8    4
```

MacSpice 2 -> print pm0

```
--------------------------------------------------------------------------------
Index   pm0                                 Numb Harmonic
--------------------------------------------------------------------------------
1       3.424668e-12,   2.480519e-01    2    1
5       -5.93273e-12,   6.485047e-04    6    3
```

The values from the tables match the waveform above. They can now be four quadrant multiplied where the cosine of the carrier is doing mainly AM and the sine of the carrier is doing mainly PM.

+/- 0.25 radian PM modulation =>

```
( .985   +0.0155cos(2wmt)  +.00002cos(4wmt))*cos(wct)      Mainly AM
(        +0.248sin(1wmt)   +.0006sin(3wmt) )*sin(wct)      Mainly PM
```

The following sinewave relationships will translate the spectrum into just three sine waves.

```
cos(A)*cos(B)  = cos(A-B)/2  + cos(A+B)/2
sin(A)*sin(B)  = cos(A-B)/2  - cos(A+B)/2
sin(A)*cos(B)  = sin(A-B)/2  + sin(A+B)/2
```

+/- 0.25 radian PM modulation => 0.985cos(wct) +0.124sin((wc-wm)t) + 0.124sin((wc+wm)t)

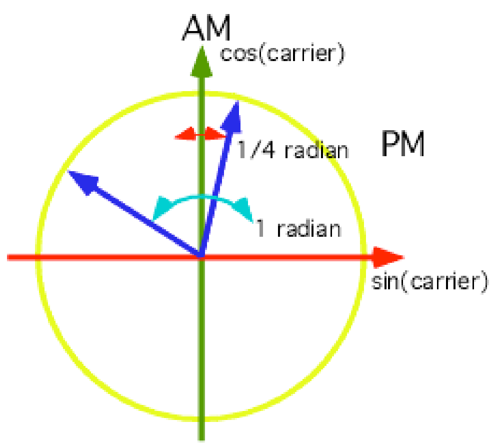
Notice the three terms are close to what a bessel function predicts as sidebands.

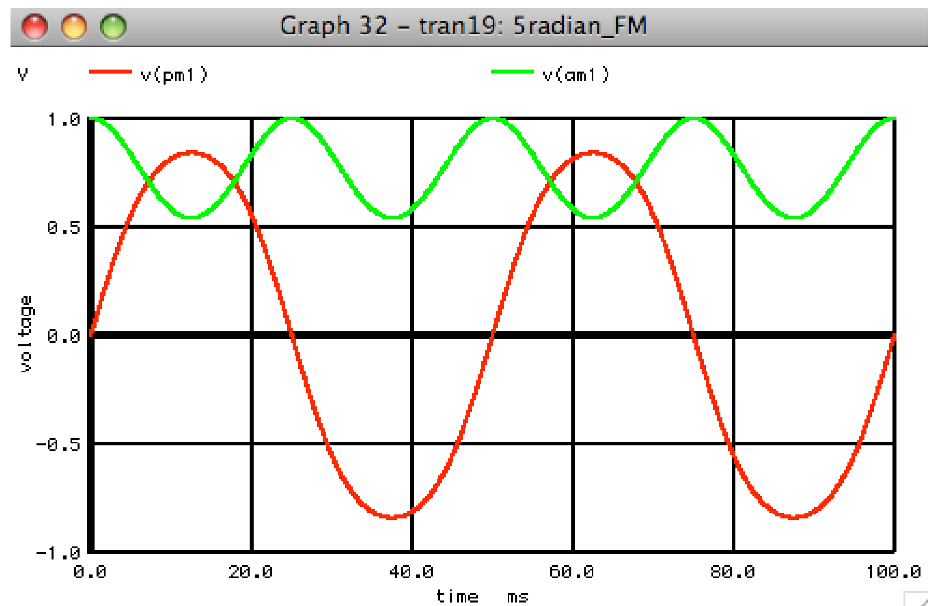
Bessel functions

The carrier and sideband amplitudes are illustrated for different modulation indices of FM signals.

Modulation index	Carrier	1	2	3	4	5	6	7	8	9	10	11	12	13
0.00	1.00													
0.25	0.98	0.12												
0.5	0.94	0.24	0.03											
1.0	0.77	0.44	0.11	0.02										

The same can now be done for one radian of phase modulation.





In this case the FFT for the am1 and pm1 signals yield as follows.

```
mean(am1) = 7.653857e-01
mean(pm1) = 1.243076e-07

--------------------------------------------------------------------------------
Index    am1                                Numb Harmonic
--------------------------------------------------------------------------------
3        2.298072e-01,    4.793574e-10     4    2
7        4.953312e-03,    1.295340e-10     8    4
11       4.187726e-05,    3.186473e-12     12   6


--------------------------------------------------------------------------------
Index    pm1                                Numb Harmonic
--------------------------------------------------------------------------------
1        -2.02133e-10,    8.801012e-01       .44
5        -3.71903e-10,    3.912684e-02       .02
9        -2.80503e-11,    4.995214e-04
13       -4.51003e-13,    3.004728e-06
```

There is now much more amplitude modulation, and the phase modulation looks much more distorted.

```
+/- 1 radian PM modulation =>
( 0.765 +0.230cos(2wmt)+.005cos(4wmt))*cos(wct)  =>       Still Mainly AM
(       +0.880sin(1wmt)+.049sin(3wmt))*sin(wct)  =>       Still Mainly PM
```

The sidebands can be express as follows.

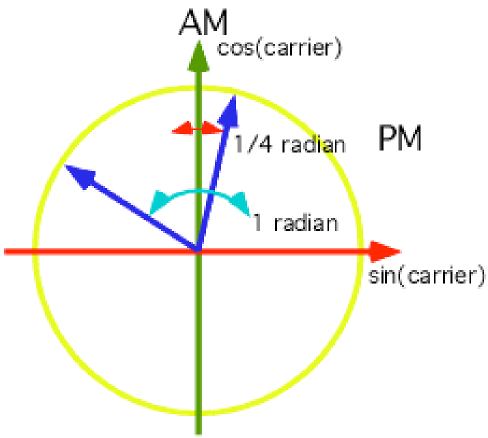
```
+/- 1 radian PM modulation =>
0.765cos(wct) + 0.440sin((wc+/-1wm)t)+ 0.115cos((wc+/-2wm)t) +0.024sin((wc+/-3wm)t)
```

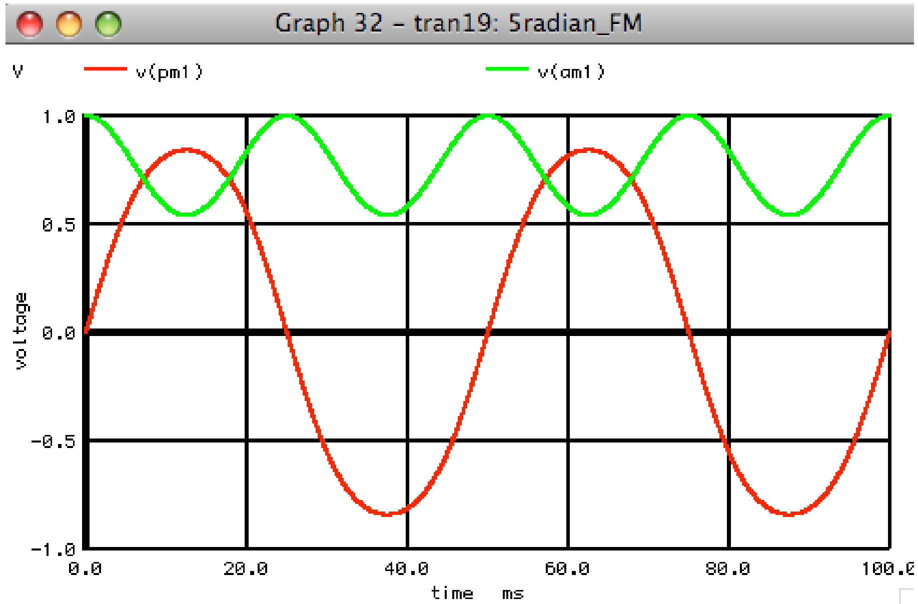
The terms match what is predicted by bessel.

Modulation index	Carrier	1	2	3	4
0.00	1.00				
0.25	0.98	0.12			
0.5	0.94	0.24	0.03		
1.0	0.77	0.44	0.11	0.02	

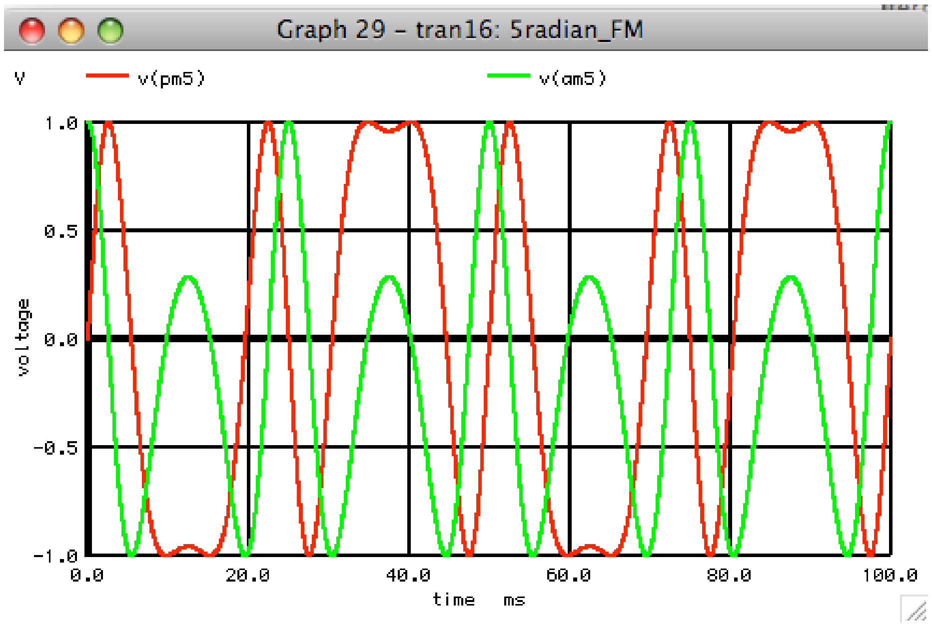
The waveforms of am1 and pm1 show where the harmonics are coming from. The amplitude is being modulated by a second harmonic while the

phase is being symmetrically distorted.





This technique appears to work well beyond one radian of modulation. At +/- 5 radians, both am5 and pm5 are getting pretty messy.



This is because the polar to rectangular conversion is really only valid over +/-90 degrees of phase. Beyond 90 degrees, x and y values begin to fold back on themselves. At +/- 5 radians, that is more than +/- 270 degrees.

mean(am5) = **-1.76657e-01**
mean(pm5) = 6.215360e-07

Index	pm5		Numb	Harmonic
1	-4.74481e-10,	**-6.55158e-01**	2	1
5	-1.83003e-08,	**7.296680e-01**	6	3
9	-3.56714e-08,	**5.222883e-01**	10	5
13	-1.94745e-08,	**1.067557e-01**	14	7
17	-4.35748e-09,	**1.104106e-02**	18	9

Index	am5		Numb	Harmonic
3	**9.313320e-02**,	7.393182e-09	4	2
7	**7.824722e-01**,	3.061402e-08	8	4
11	**2.621026e-01**,	3.009379e-08	12	6
15	**3.681173e-02**,	1.007368e-08	16	8
19	**2.935766e-03**,	1.561527e-09	20	10

There are a lot more terms that now need to be added to the spectrum.

+/- 5 radian PM modulation =>

```
( -.177 +0.093cos(2wmt)+.782cos(4wmt)+.262cos(6wmt)+.036cos(8wmt))*cos(wct)
(       -0.655sin(1wmt)+.729sin(3wmt)+.522sin(5wmt)+.104sin(7wmt))*sin(wct)
```

The total spectrum can now be arranged to follow the terms predicted by bessel.

+/- 5 radian PM modulation =>

```
=> -.177cos(wct)+0.327sin((wc+/-1wm)t)+0.045cos((wc+/-2wm)t)+0.364sin((wc+/-3wm)t)+0.391cos((wc+/-4wm)t)
   +0.261sin((wc+/-5wm)t)+0.131cos((wc+/-6wm)t)+0.052sin((wc+/-7wm)t)+0.018cos((wc+/-8wm)t)
```

Modulation index	Carrier	1	2	3	4	5	6	7	8	
0.00	1.00									
0.25	0.98	0.12								
0.5	0.94	0.24	0.03							
1.0	0.77	0.44	0.11	0.02						
1.5	0.51	0.56	0.23	0.06	0.01					
2.0	0.22	0.58	0.35	0.13	0.03					
2.41	0	0.52	0.43	0.20	0.06	0.02				
2.5	−.05	0.50	0.45	0.22	0.07	0.02	0.01			
3.0	−.26	0.34	0.49	0.31	0.13	0.04	0.01			
4.0	−.40	−.07	0.36	0.43	0.28	0.13	0.05	0.02		
5.0	−.18	−.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02	

What this suggests is that any nonlinear type of FM can be converted to a full spectrum complete with magnitude and phase. There is always a one to one relationship of Phase modulation to frequency modulation in that phase shift always needs a frequency shift to precede it. But there may not any limitations on type of waveshape or amplitude or type of modulation on the wave form when it comes to finding its spectrum.