

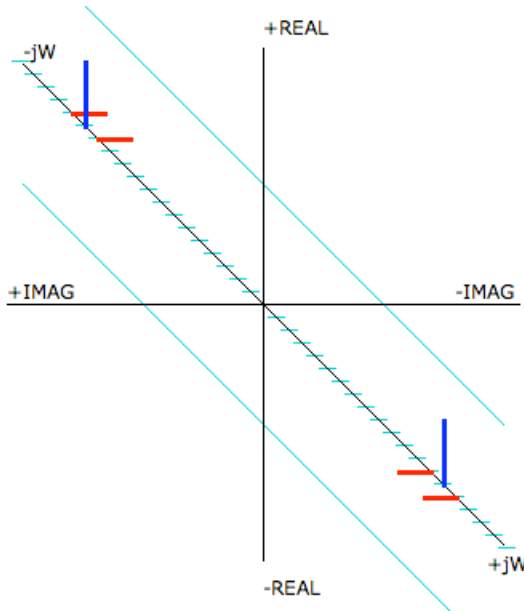
=====EULER\_Spectrum\_Format=====

There appears to be several different styles of format for spectrum analysis. Perhaps the most precise format will translate according to the Euler identities shown below.

$$\begin{aligned} \text{Cos}(x) &= \exp(j*x)/2 + \exp(-j*x)/2 \\ \text{Sin}(x) &= \exp(j*x)/2*j - \exp(-j*x)/2*j \end{aligned}$$

=====EULER\_Format=====

Some details need to be taken into account when using this format.



First, frequency is being used in terms of radians per second which is often called "j\*w0". The relationship is as such.

$$j*x = "j*w0" = 2*Pi*freq\_Hz$$

Second, a single frequency is represented by a pair of elements placed symmetrically around the +/- range of a "j\*w" dimension.

Third, these two elements both have an equal magnitude.

Fourth, the magnitudes can have a complex value which represents phase.

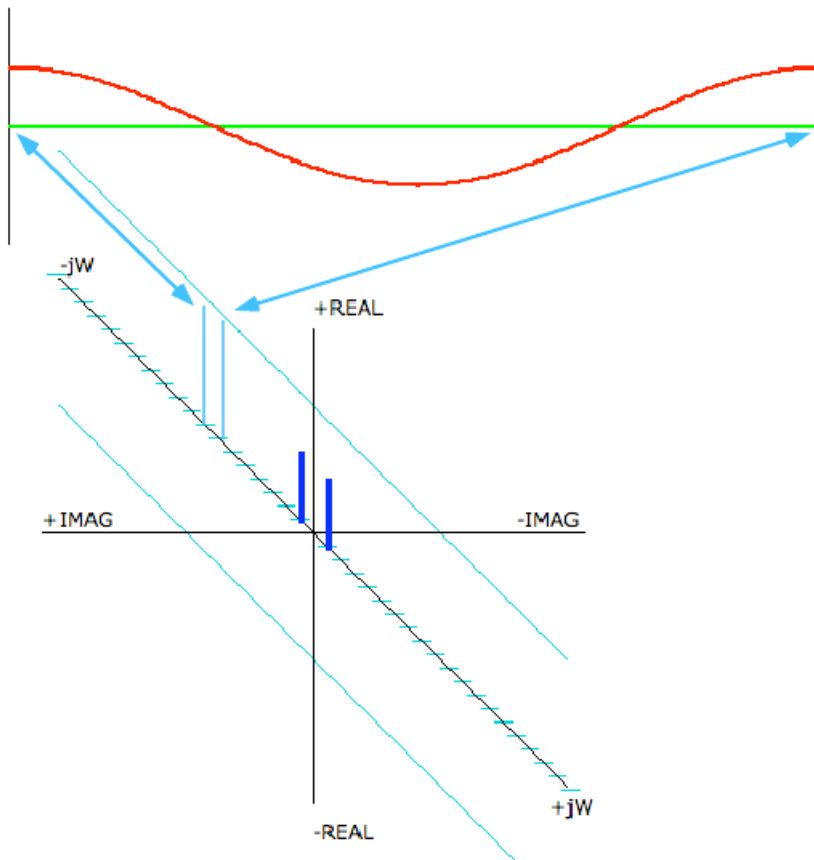
Fifth, the magnitudes at -j\*w or not a mirror image of that at +j\*w. The image is more like a TV camera image.

Sixth, the spectrum is in terms of magnitudes rather than power. Some care must therefore be taken when dealing in terms of RMS.

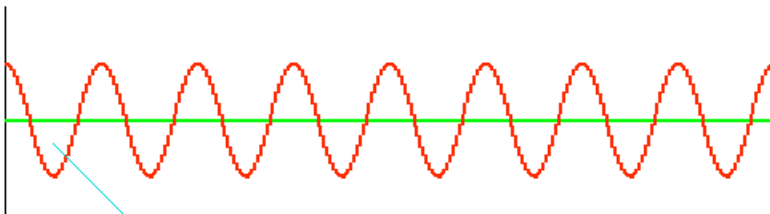
=====FFT\_details=====

Plotting a FFT result as Euler identity in 3D can show everything. But first a few details about FFT data.

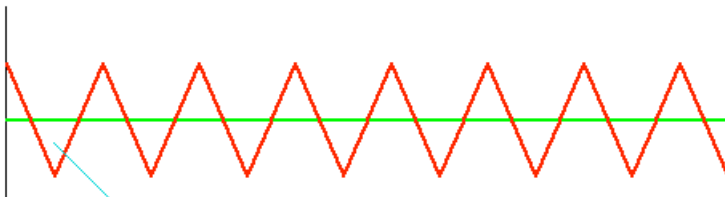
First, the **total sample period** defines **frequency Bin resolution**. One cannot honestly know whether a 1MegHz sine-wave has undergone Amplitude or Phase Modulation at a 1Hz rate until the 1MegHz signal has been measured for 1 whole second.



Second, having signals that have **whole units of cycles** contained within the full sample period makes for much sharper spectrums. Otherwise a "windowing" function may have to be applied.

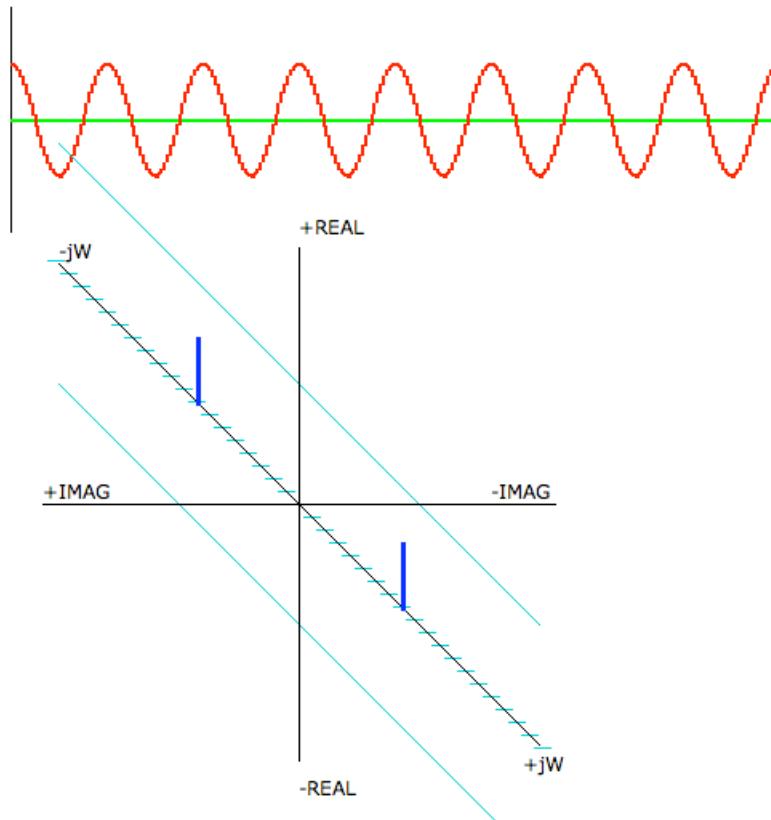


Third, one needs at **least two points to see any AC**. To see a signal that has exactly **8 cycles** within the full sample period, one is going to need to take at least **16 samples**. Eight samples can detect when the 8 cycle signal is high and the other alternating eight samples can detect when the 8 cycle signal is low. So the Nyquist frequency is half the sample rate.

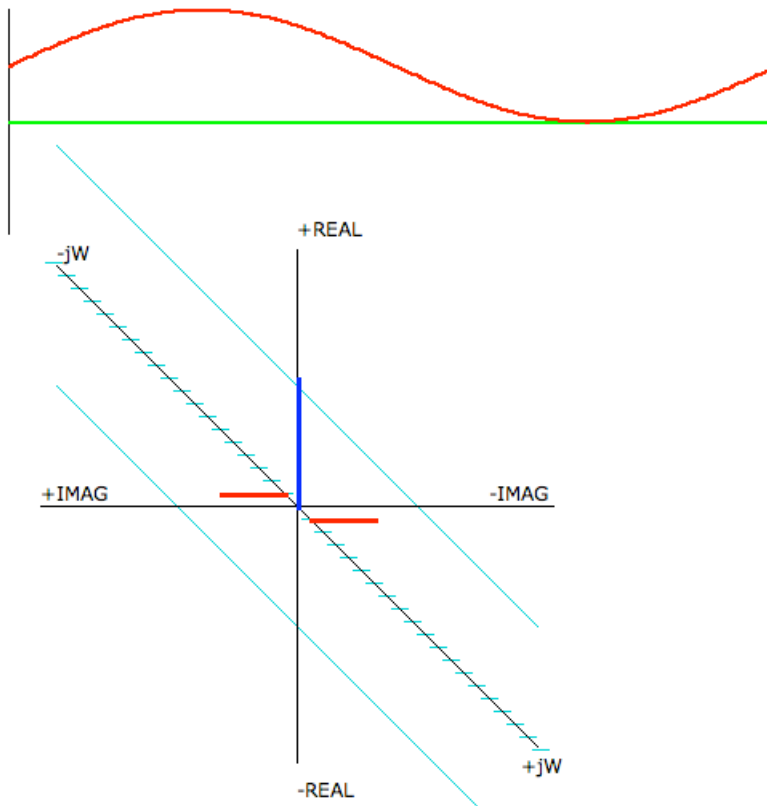


Fourth, the FFT is going to rearrange the signal into "bins". **Each bin is just a multiple of the frequency bin resolution**. For instance, one full second of total sample time corresponds to a 1Hz bin resolution. Therefore any 1Hz input signal will show up in bin number one. And any 3Hz input signal shows up in bin

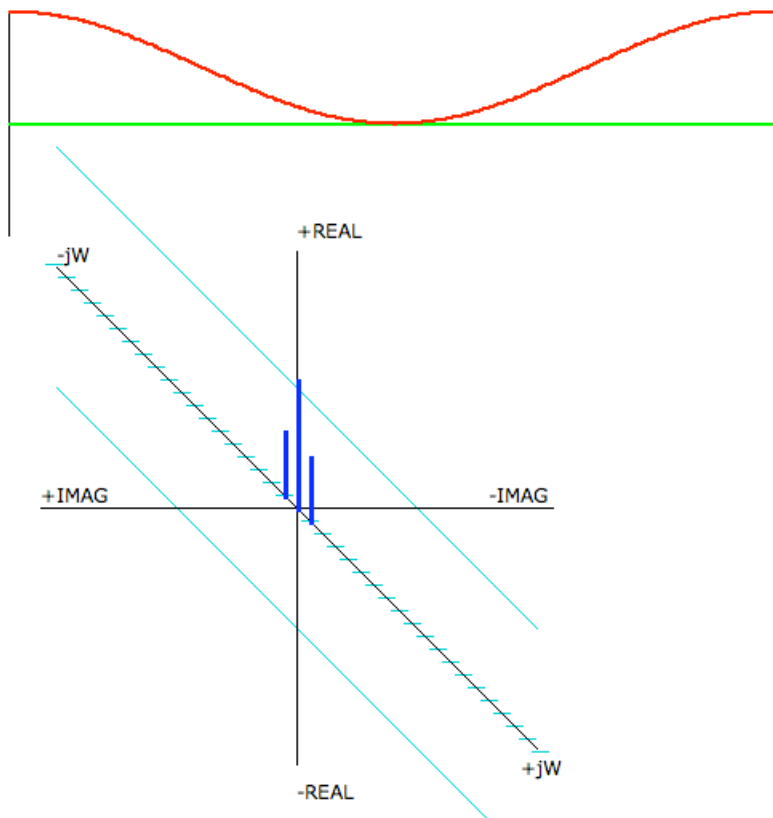
number three.



Fifth, the value stored within **the bin can be complex**. That means it stores both a magnitude and phase for each frequency point (Bin Number). Sine waves are defined in terms of magnitude, frequency, and phase.



Sixth, the FFT actually contains a DC point at bin number zero. The value in bin zero is real as is the value at the last bin. The FFT can take 16 samples of data and rearrange them into 16 other locations. Bin zero contains one DC data point (real). Bins one to seven each contain two data points (complex). The last bin (nyquist) can only contain one data point (real).

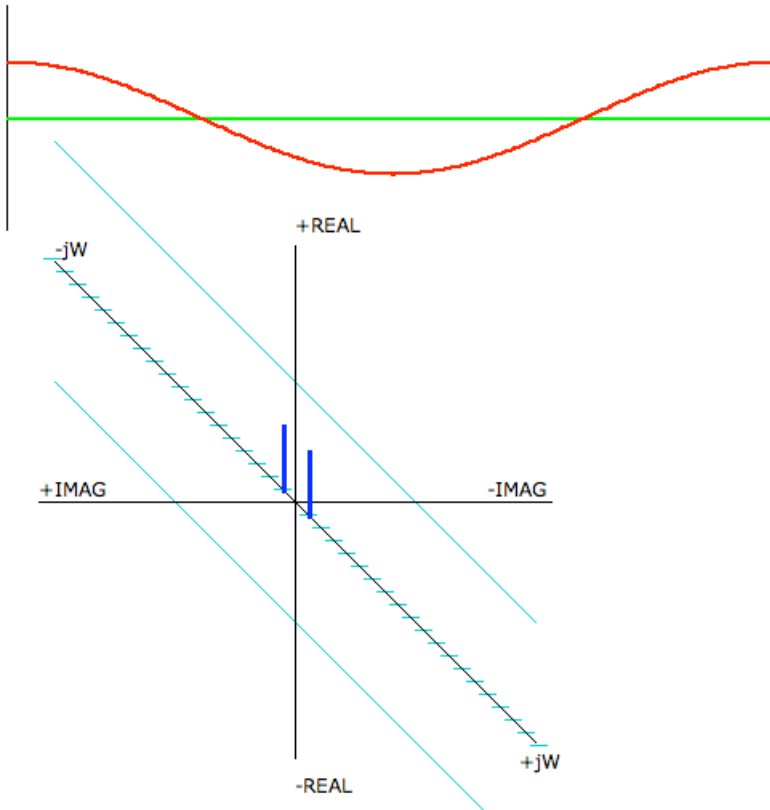


### =====FFT\_End\_Point\_Details=====

It may be easiest to just think in terms of what kind of frequency is desired. If one has a 1kHz signal which is sampled over 1msec, then the frequency resolution of each bin will be 1KHz. The fundamental will show up in bin number one. The second and third harmonics will show up in bin number two and three.

A unit magnitude cosine wave for a 1KHz signal is shown below.

$$V_{in} = \cos(\omega t_0 * T) = \cos(2 * \pi * 1\text{KHz} * T) \quad 0 \rightarrow T \rightarrow 1\text{msec}$$



A cosine wave shows up as two elements located at +/- bin number "one" on the 3D graph.

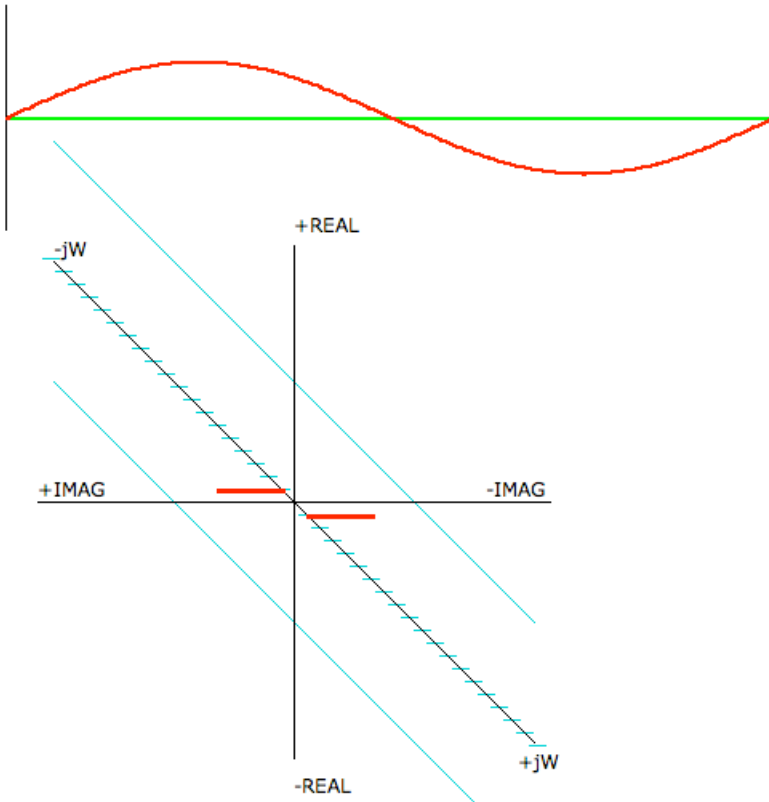
wt/wt_0	Real	Imaginary
0	0.000	*****
1	0.5000	0.000
2	0.000	0.000
etc...		

The cosine is just repackaged and graphed in Euler identity format.

$$\text{Cos}(wt_0*T) = \exp(j*1*wt_0*T)/2 + \exp(-j*1*wt_0*T)/2$$

The complex feature allows phase information to be stored as well. A unit magnitude sine wave for a 1KHz signal is shown below.

$$V_{in} = \sin(wt_0*T) = \sin(2*pi*1KHz*T) \quad 0 \rightarrow T \rightarrow 1\text{msec}$$

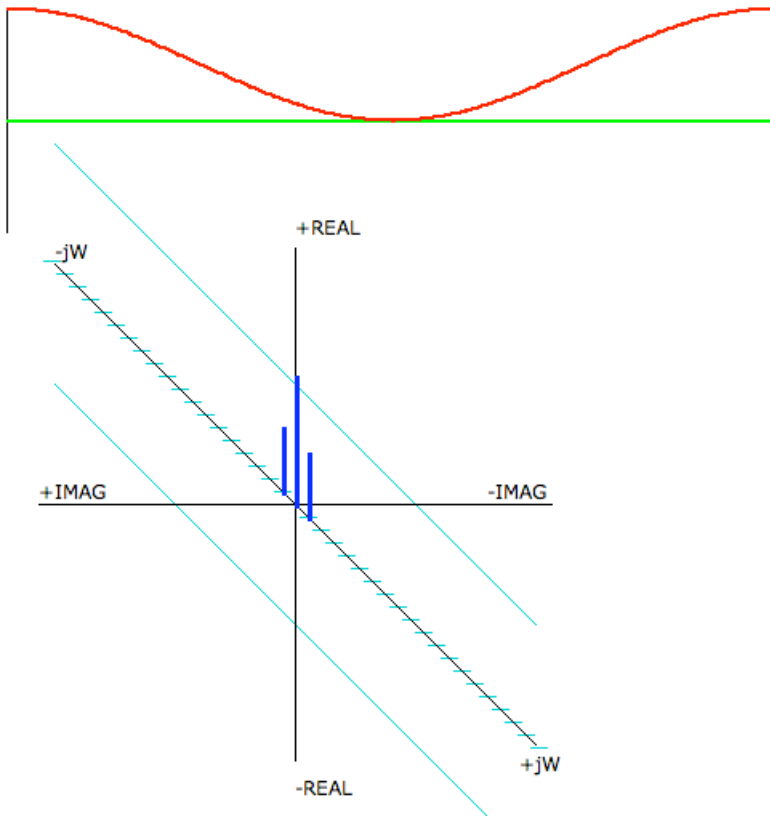


The sine wave is just repackaged and graphed in Euler identity format.

$$\sin(\omega t_0 * T) = \frac{\exp(j * 1 * \omega t_0 * T)}{2 * j} - \frac{\exp(-j * 1 * \omega t_0 * T)}{2 * j} - j * \frac{\exp(j * 1 * \omega t_0 * T)}{2} + j * \frac{\exp(-j * 1 * \omega t_0 * T)}{2}$$

wt/wt_0	Real	Imaginary
0	0.000	*****
1	0.000	-0.5000
2	0.000	0.000
3	0.000	0.000
etc...		

The DC term can be thought of what happens when you look at a cosine with a very low frequency response. It will look like a DC of one. A unity cosine together with a unity DC term is shown below.



wt/wt_0	Real	Imaginary
0	1.000	*****
1	0.5000	0.000
2	0.000	0.000
etc...		

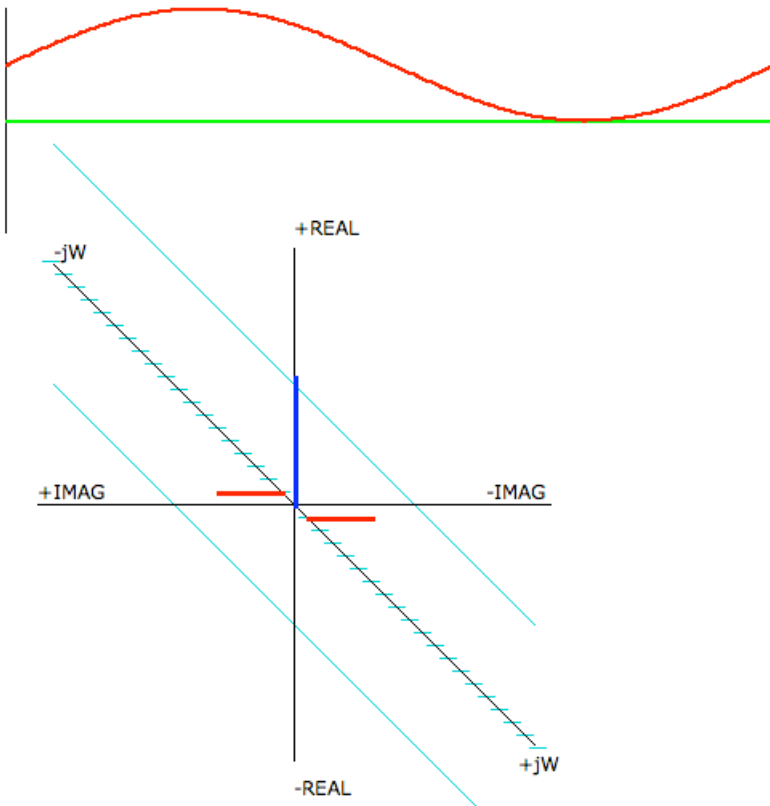
Imagine the two 1/2 AC terms coming together to form unity.

The DC term is just two 1/2 magnitude real terms summing at the origin.

$$1 = \exp(j*0*wt_0*T)/2 + \exp(-j*0*wt_0*T)/2$$

A very low frequency unity sine wave will appear as zero.

The graph below shows a unity sine wave with a unity DC term.



Since the imaginary terms must always cancel out at the origin, it is never possible to have an imaginary term at **bin number zero**.

$$0 = \exp(j*0*wt_0*T)/2*j - \exp(-j*0*wt_0*T)/2*j$$

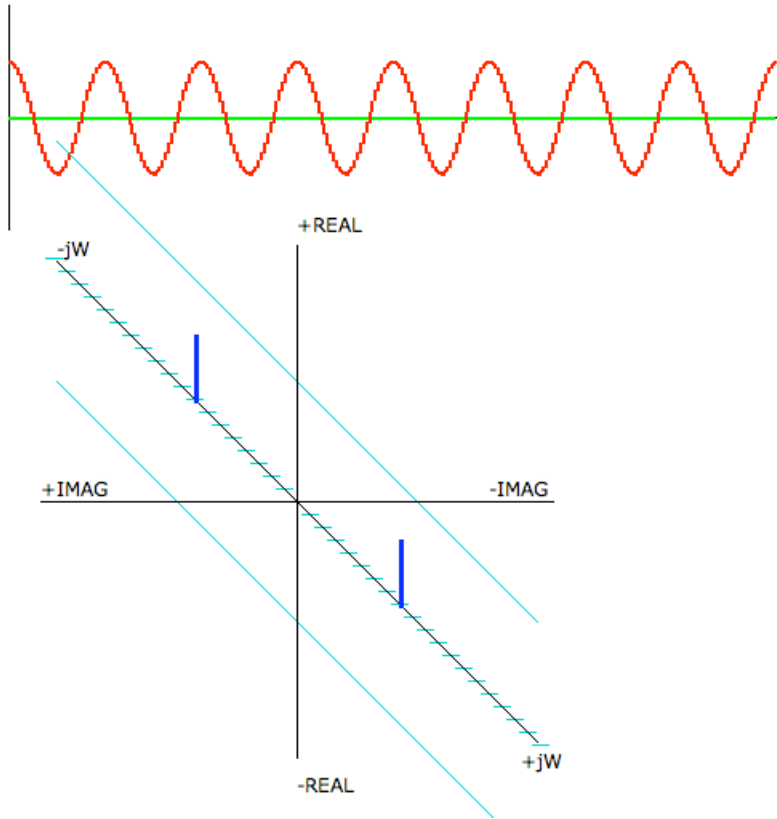
wt/wt_0	Real	Imaginary
0	1.000	*****
1	0.000	-0.5000
2	0.000	0.000
etc...		

At the other end of the spectrum, it turns out that the Nyquist term is also appears to be constrained to be real.

Consider applying eight cycles of cosine to look like so.

$$V_{in} = \cos(8*wt_0*T) = \cos(2*pi*1KHz*8*T) \quad 0 \rightarrow T \rightarrow 1msec$$





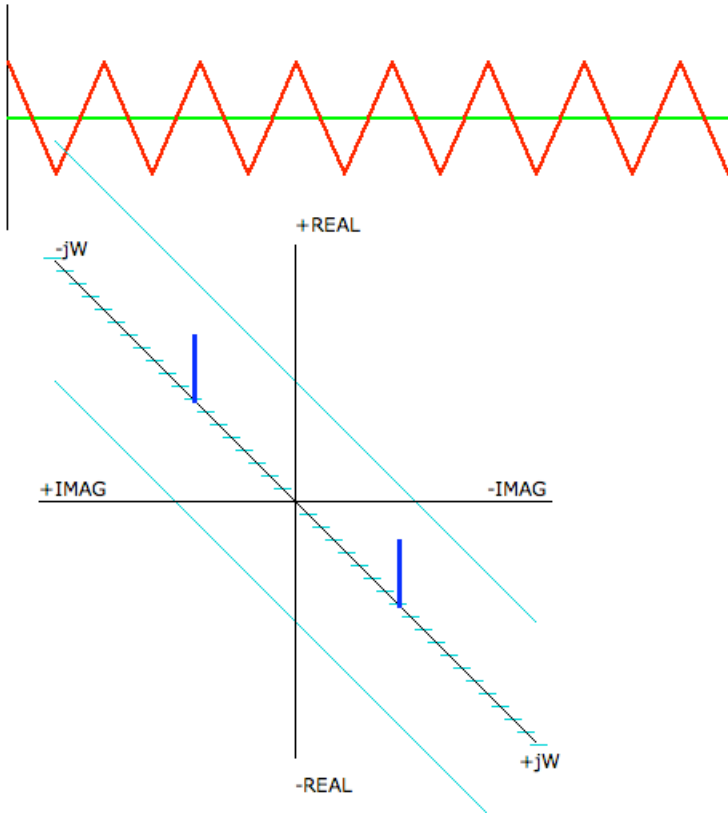
With 256 samples, the cosine wave is visible at bin number eight.

```

N=256
wt/wt_0   Real       Imaginary
0         0.000     *****
1         0.000     0.000
2         0.000     0.000
3         0.000     0.000
4         0.000     0.000
5         0.000     0.000
6         0.000     0.000
7         0.000     0.000
8         0.500     0.000
9         0.000     0.000
etc...

```

Now reduce the total number of samples to 16.  
 Since it is a cosine wave, the samples will alternate  
 between the cosine wave's high and low value.

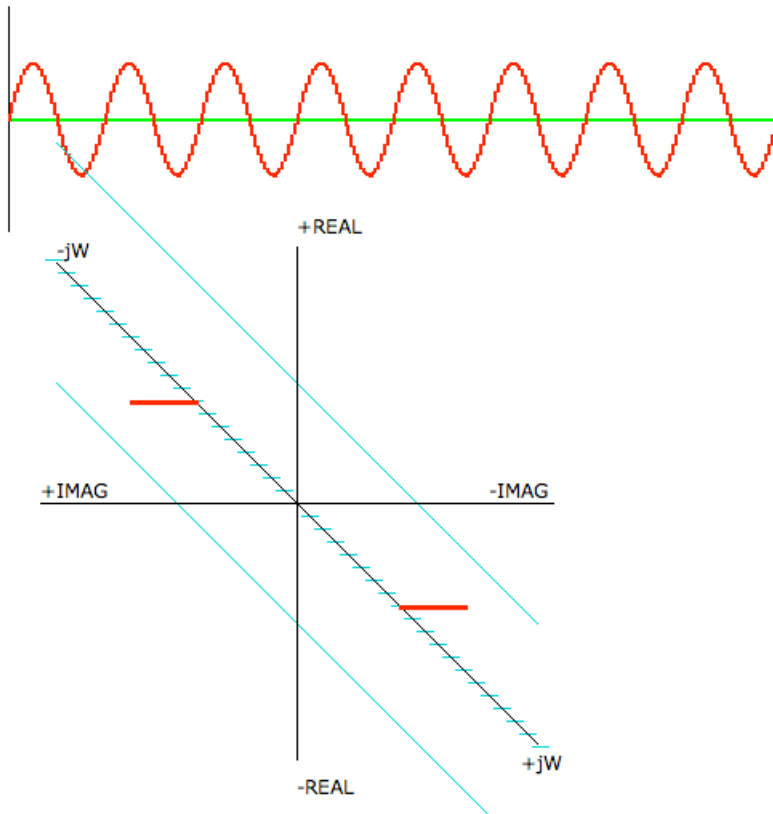


Exactly at Nyquist the FFT results should still be the same.  
 (Writing your own FFT code to do this correctly might be tricky.)

```

N=16
wt/wt_0   Real       Imaginary
0         0.000      *****
1         0.000      0.000
2         0.000      0.000
3         0.000      0.000
4         0.000      0.000
5         0.000      0.000
6         0.000      0.000
7         0.000      0.000
8         0.5000     ?????
  
```

What about for a sine wave?



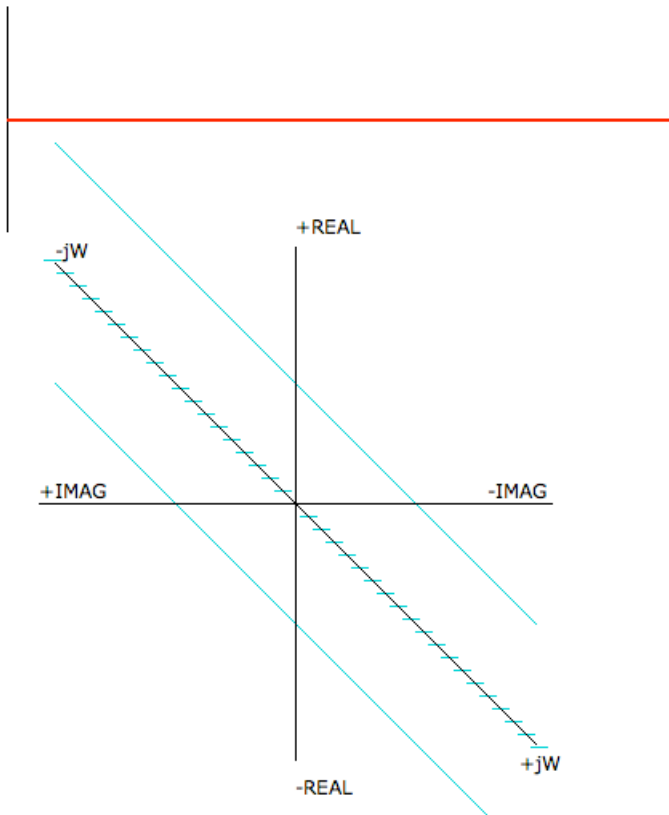
With 256 samples, the sine wave is visible at bin number eight.

```

N=256
wt/wt_0  Real      Imaginary
0        0.000    *****
1        0.000    0.000
2        0.000    0.000
3        0.000    0.000
4        0.000    0.000
5        0.000    0.000
6        0.000    0.000
7        0.000    0.000
8       0.000    -0.5000
9        0.000    0.000
10       0.000    0.000
11       0.000    0.000
etc...

```

Now reduce the total number of samples to 16.  
 Since it is a sine wave, all samples will happen at a zero crossing



```

N=16
wt/wt_0  Real      Imaginary
0         0.000    *****
1         0.000    0.000
2         0.000    0.000
3         0.000    0.000
4         0.000    0.000
5         0.000    0.000
6         0.000    0.000
7         0.000    0.000
8         0.000    *****

```

**So is it possible to have an imaginary term for a sine wave at nyquist? If not, that means that 16 data points got rearranged into 16 different values and locations. Getting anything other than 16 data points out of 16 input points might raise some questions.**

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