Alternative View of Frequency Modulation

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When a spectrum analysis is done on a FM signal, a odd set of side bands show up. This suggests that the Frequency modulation is a very nonlinear modulation process. The fact that the side bands are following a Bessel function of the first kind is not making things any more intuitive. A very good tutorial in Frequency Modulation can be found at the following site. A clip from that tutorial is just below it.

http://www.techonline.com/learning/techpaper/208400860

β	J0	J1	J2	J3	J4	J5	J6	J7	J8
0	1								
0.25	0.98	0.12							
0.5	0.94	0.24	0.03						
1.0	0.77	0.44	0.11	0.02					
2.0	0.22	0.58	0.35	0.13	0.03				
3.0	-0.26	0.34	0.49	0.31	0.13	0.04	0.01		
4.0	-0.40	-0.07	0.36	0.43	0.28	0.13	0.05	0.02	
5.0	-0.18	-0.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02

Table 1. Bessel Functions of the First Kind Rounded to Two Decimal Places.

If $A_c^2/2 = 1$, β=1, f_m = 1 kHz, and f_c = 100 kHz, then the result is the FM voltage spectrum shown in Figure 2

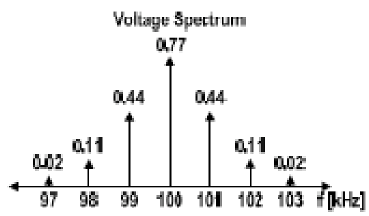


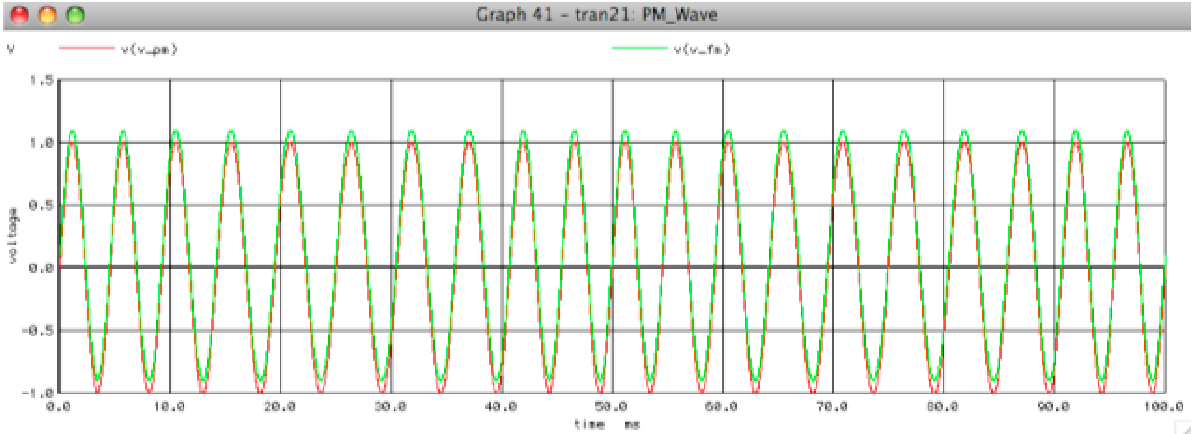
Figure 2. FM Voltage Spectrum for $\frac{A_c^2}{2} = 1$, β=1, fm = 1 kHz, and fc = 100 kHz.

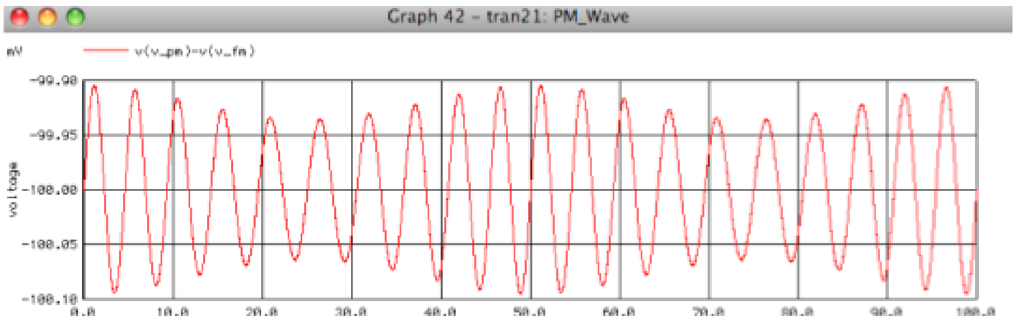
FM might be thought of as being both linear and nonlinear depending on the point of view. FM and PM appear to be some what equivalent in many ways. The following web site shows that if a carrier signal gets frequency modulated by the same frequency twice, it will come be somewhat equivalent to a carrier being phase modulated by that same frequency at a 1 radian peak phase amplitude.

http://www.idea2ic.com/PlayWithSpice/pdf/Tranlate_between_PM_FM.pdf

The only difference being a 90 degree phase difference between the signal modulating the FM as oppose to the PM. Or to say it another way, take any carrier frequency. If you use a 1Khz to Phase modulate it at a +/- 1 radian magnitude, the PM signal will be equivalent to Frequency modulating the same carrier by a 1kHz signal to a 1kHz peak frequency deviation. The FM tutorial states that the ratio of frequency deviation to message signal frequency is called the modulation index, same as is described in the Spice's single frequency model. It just so happens that this modulation index is also just the number of radians of phase modulation if the FM signal is thought of in terms of being Phase modulated instead. This appears to bear out in terms of a spice simulation given below.

http://www.idea2ic.com/PlayWithSpice/pdf/PM_is_FM.pdf



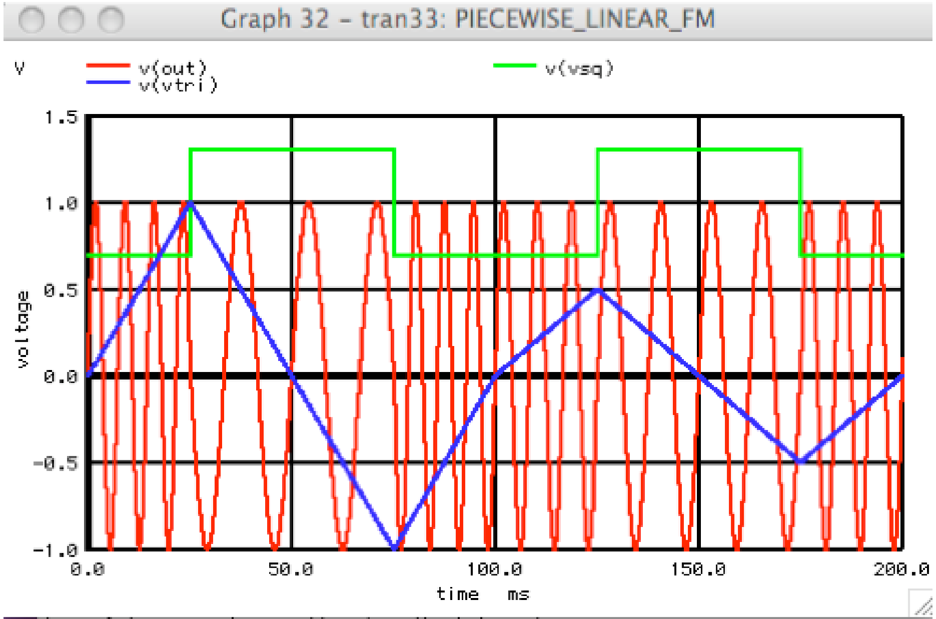


To a certain extent the comparison simulations were done with some inaccuracies to verify that two different signals are really being compared. Charles D. H. Williams (MacSpice Author) sent me some notes as to how to greatly improve the simulation accuracy. The less that accurate comparisons between FM and PM is given at previous listed web site.

Knowing that the "modulation index" represents really just the peak phase modulation seems to be more intuitive. For instance when Magnavox proposed a AM Stereo system which had a +/- 5radian 5Hz pilot signal, that corresponds to a "modulation index" of 5 where the AM station's carrier is being frequency modulated by 5Hz at a 25Hz peak frequency deviation.

During the development of the LM1981 (the first AM Stereo decoder), it was discovered that the translation between FM and PM could be done completely at audio. For AM Stereo, both PM and FM detection could be done for the Magnavox system by partially integrating a FM decoded signal. The page given below shows how to generate a FM signal by integrating the message signal before applying it to a Phase Modulator.

http://www.idea2ic.com/PlayWithSpice/pdf/FM_PWL.pdf

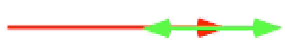


This might make it easier inn Spice to generate the type of FM signals one would expect to see coming out of a VCO.

So in terms of the PM/FM world, nothing gets done to the modulating signal which would distort it. So why it the spectrum of a FM signal so strange? It is because FM and PM signals are linear in the POLAR world while spectrum analysis is really seeing things in the RECTANGULAR world. So there is an polar to rectangular conversion taking place when FM or PM is being viewed from a sine/cosine wave point of view.

Start off with some fundamentals. Assume the carrier is a cosine wave. If a small cosine wave of the same frequency is either added or subtracted, only effective amplitude of the carrier will change. Its phase before and after will remain the same.

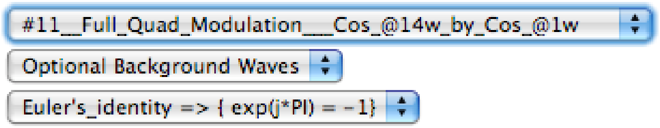
If the small added cosine is moving back and forth then the amplitude of the carrier is being modulated.



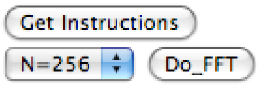
The following web site is to a javascript page that can show this kind of frequency process in action.

http://www.idea2ic.com/PlayWithJavascript/3D_FFT8v.html

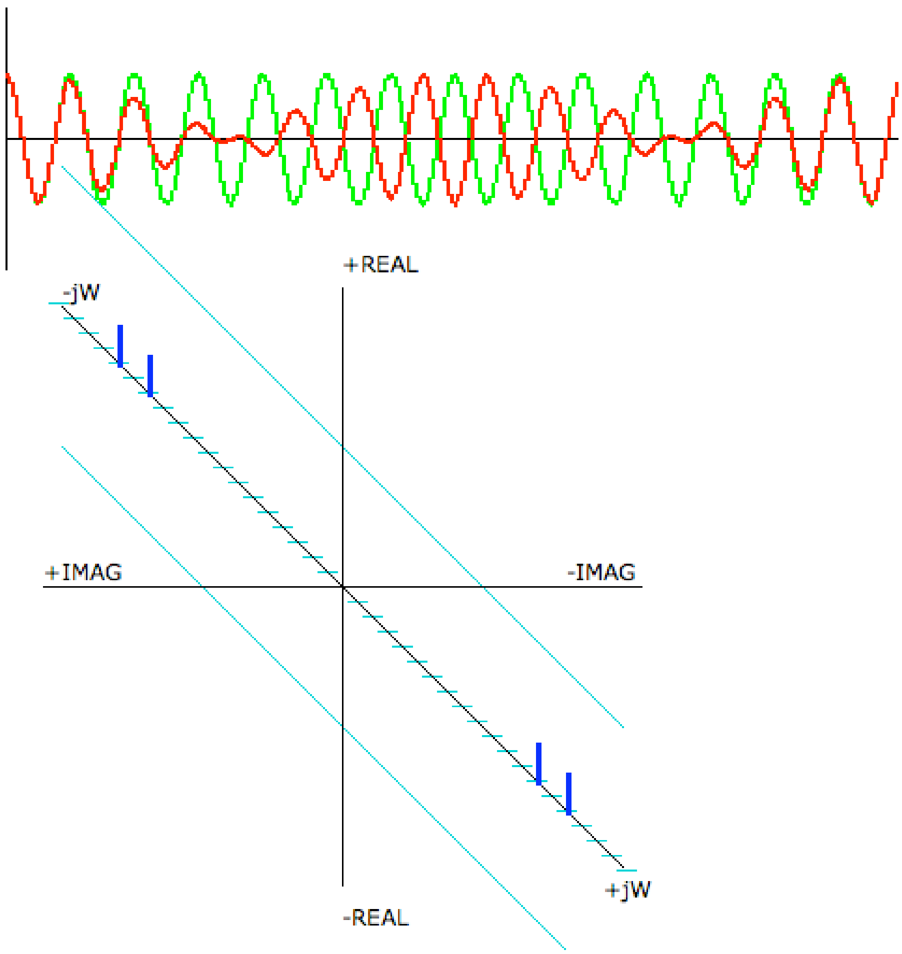
Provided a web browser like Opera is being used with it javascript feature enabled, the page is designed so that a waveform is first selected and then the "Do_FFT" button is pushed. The selected waveform is plotted over time and also in a 3D Euler Identity format. (Read the instruction if so inclined.) Notice "cosine signal" is colored blue while "sine signal" is coded red. Suppose signal #11 is selected.



After the selection the "Do_FFT" button should be pushed



The plotted waveforms should be this..



Also note that the actual outputs of the FFT are listed as well.

wt/wt_0	Real	Imaginary
0	0.000	0.000
1	0.000	0.000
2	0.000	0.000
3	0.000	0.000
4	0.000	0.000
5	0.000	0.000
6	0.000	0.000
7	0.000	0.000
8	0.000	0.000
9	0.000	0.000
10	0.000	0.000
11	0.000	0.000
12	0.000	0.000
13	0.2500	0.000
14	0.000	0.000
15	0.2500	0.000
16	0.000	0.000
17	0.000	0.000
18	0.000	0.000

The carrier signal is going through a full four quadrant modulation process which in this case is multiplying a carrier at a frequency of 14w by a modulation signal at a frequency of 1w. The result are the sum and different frequencies of 13w and 15w.

#11__Full_Quad_Modulation___Cos_@14w_by_Cos_@1w

Optional Background Waves

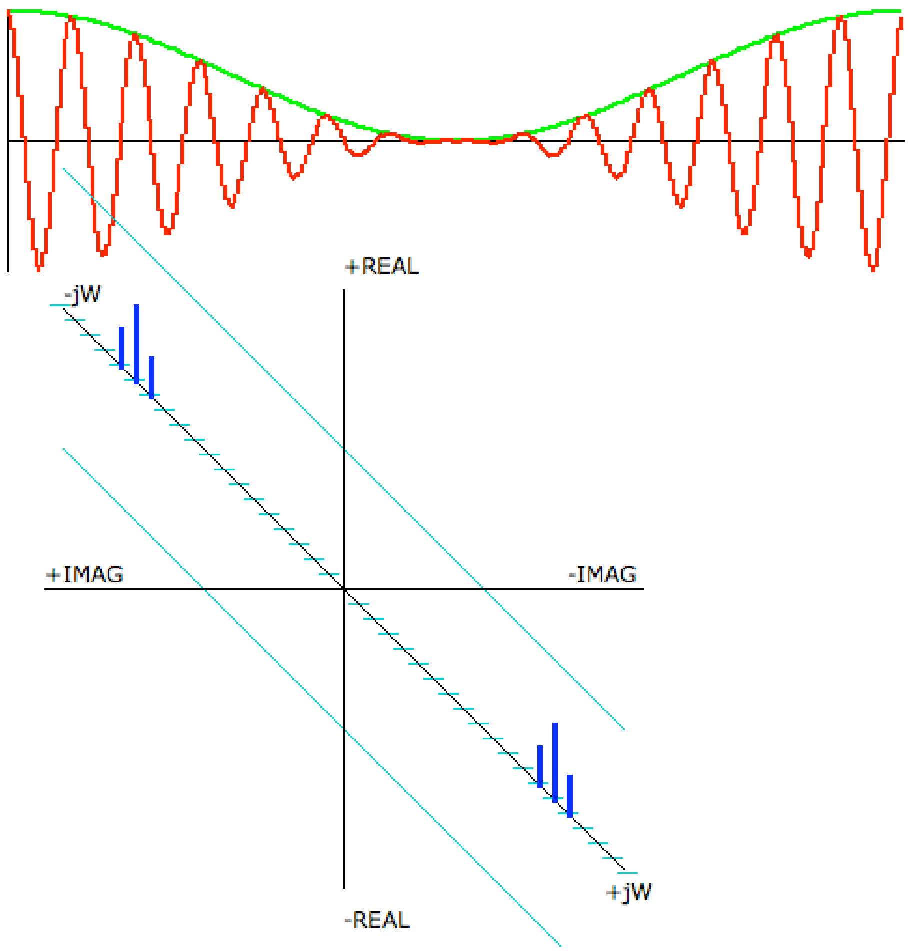
Euler's_identity => { exp(j*PI) = -1}

The sum and difference sidebands effective produce a cosine carrier at a frequency of 14w that is varying in magnitude at a 1w rate. Select waveform #7 to see how the sidebands amplitude modulate the carrier.

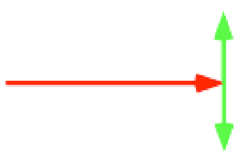
#7___Amplitude_Modulation___Cos_@14w_by_Cos_@1w

Optional Background Waves

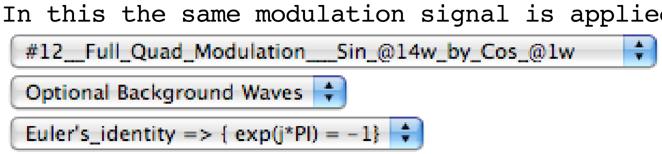
Euler's_identity => { exp(j*PI) = -1}

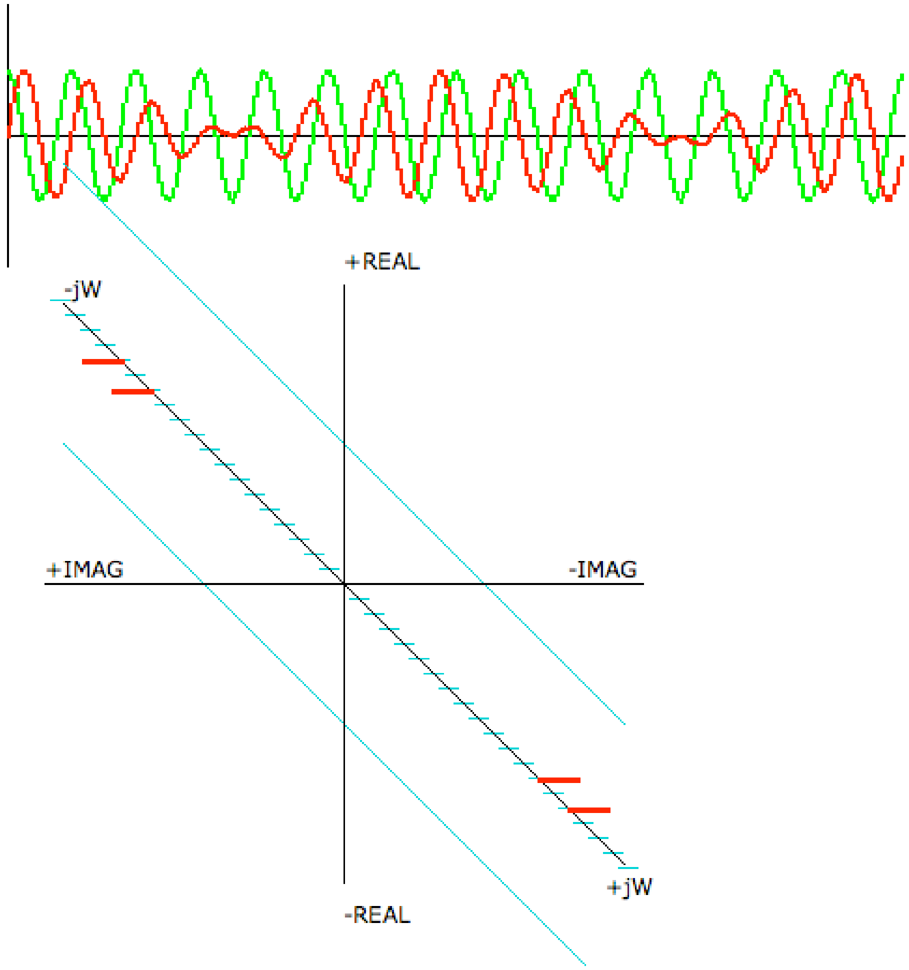


Now suppose your radio station has another carrier signal which happens to be a "sine-wave signal". If a small amount of this signal gets added to the "cosine carrier signal", then the carrier will tend to remain at the same amplitude but with vary in phase.



In this the same modulation signal is applied to a "sine carrier signal."





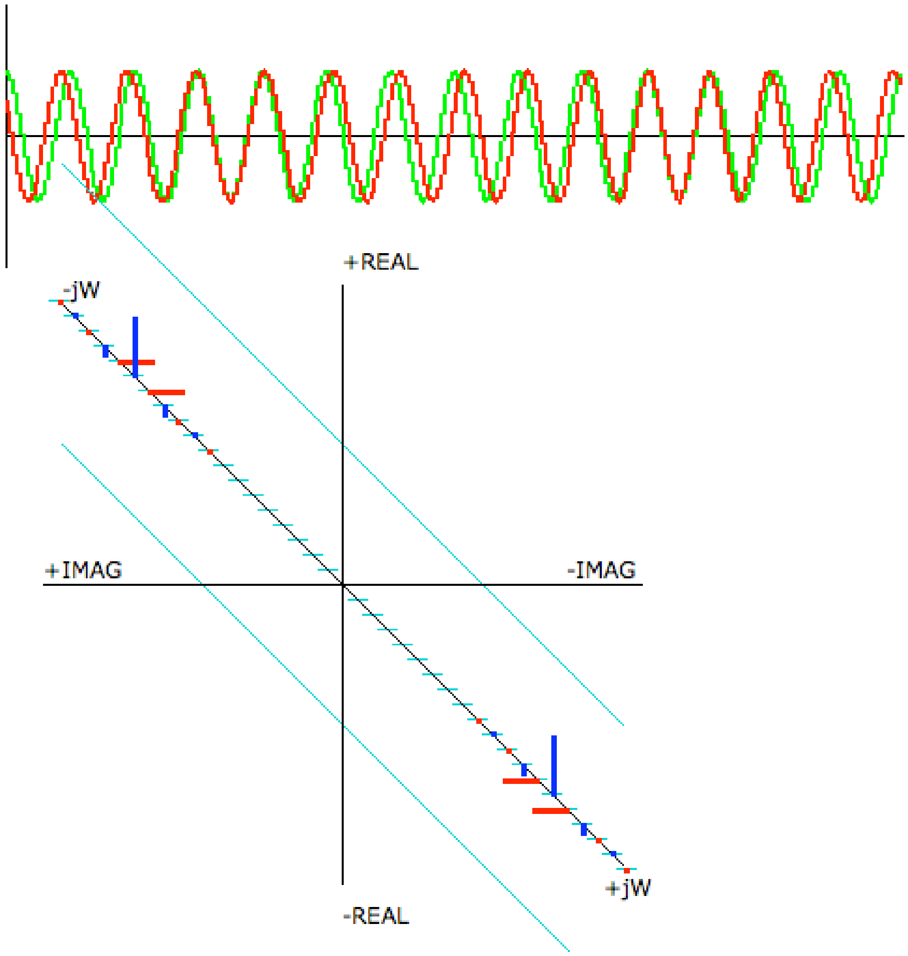
Notice both sidebands are red and that the varying carrier is always 90 degrees of phase shift from a background carrier.

Now select #9 for full 1 radian phase modulation.

#9___Phase_Modulation___Cos_@14w_by_Cos_@1w_@1rad

Optional Background Waves

Euler's_identity => { exp(j*PI) = -1}

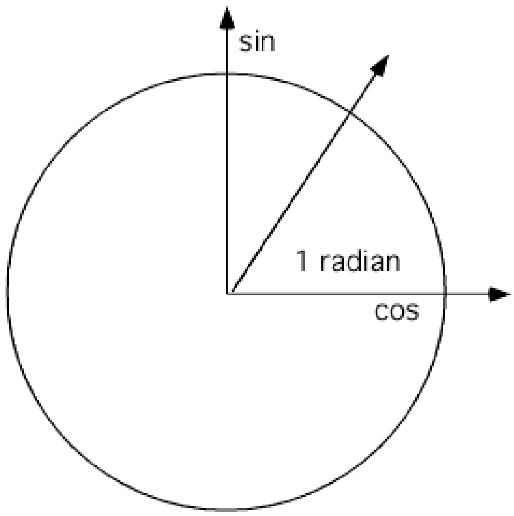


The sidebands are mainly sideways. But there are some "additional" side bands in both the AM (up and down) and PM (sideways) directions. The actual values are as follows.

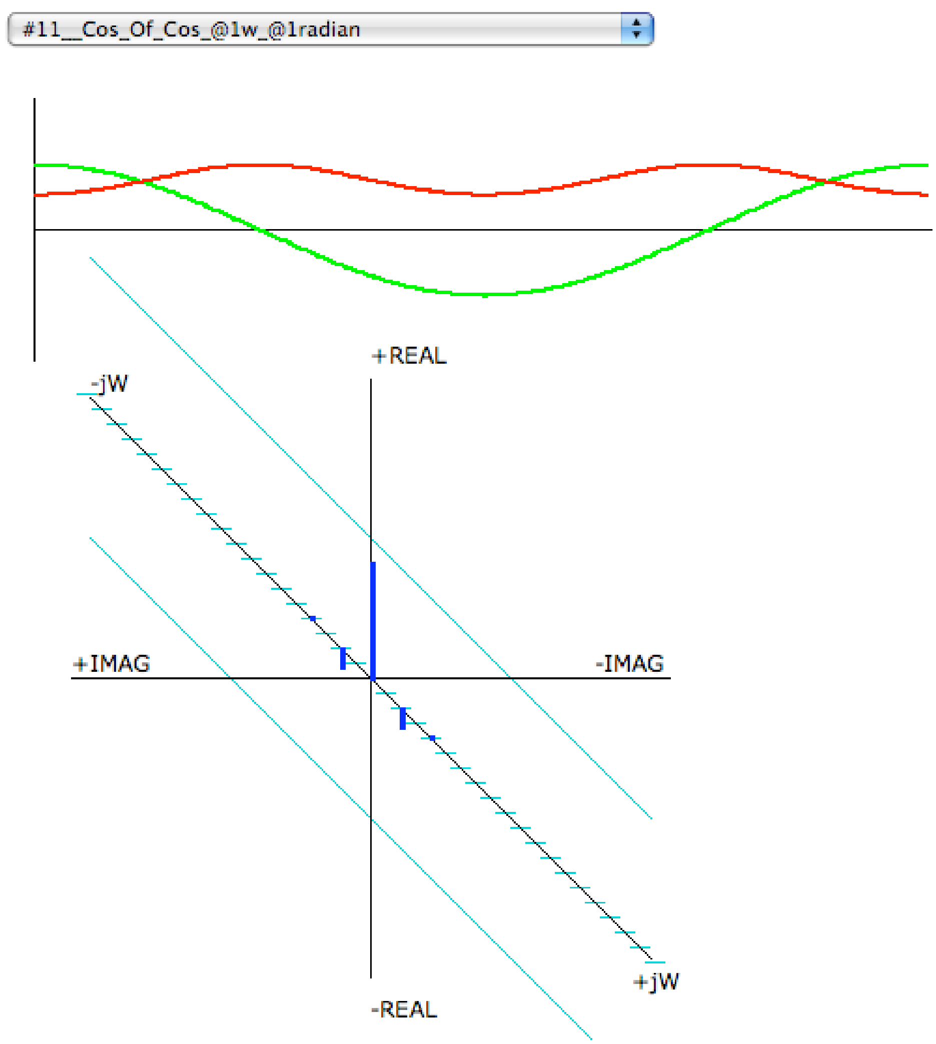
wt/wt_0	Real	Imaginary
0	0.000	0.000
1	0.000	0.000
2	0.000	0.000
3	0.000	0.000
4	0.000	0.000
5	0.000	0.000
6	0.000	0.000
7	0.000	0.000
8	0.000	0.000
9	0.000	0.0001249
10	0.001238	0.000
11	0.000	-0.009782
12	-0.05745	0.000
13	0.000	0.2200
14	0.3826	0.000
15	0.000	0.2200
16	-0.05745	0.000
17	0.000	-0.009782
18	0.001238	0.000

Since this is 1 radian of PM or modulation index of one for FM, compare the Bessel function numbers to the value of the sidebands. Just in terms of magnitude, there is only a factor of 2 difference between the two. (Because of Euler identity format).

Well if a carrier magnitude is kept constant and its phase is linearly varied, isn't that linear modulation on the polar plane? But isn't sine and cosine in the rectangular plane? One radian is not a small angle as is show below. What happens when linear modulated phase (polar) gets translated to rectangular coordinates (sin/cos)?



Start with converting the polar vector to the cosine plane.



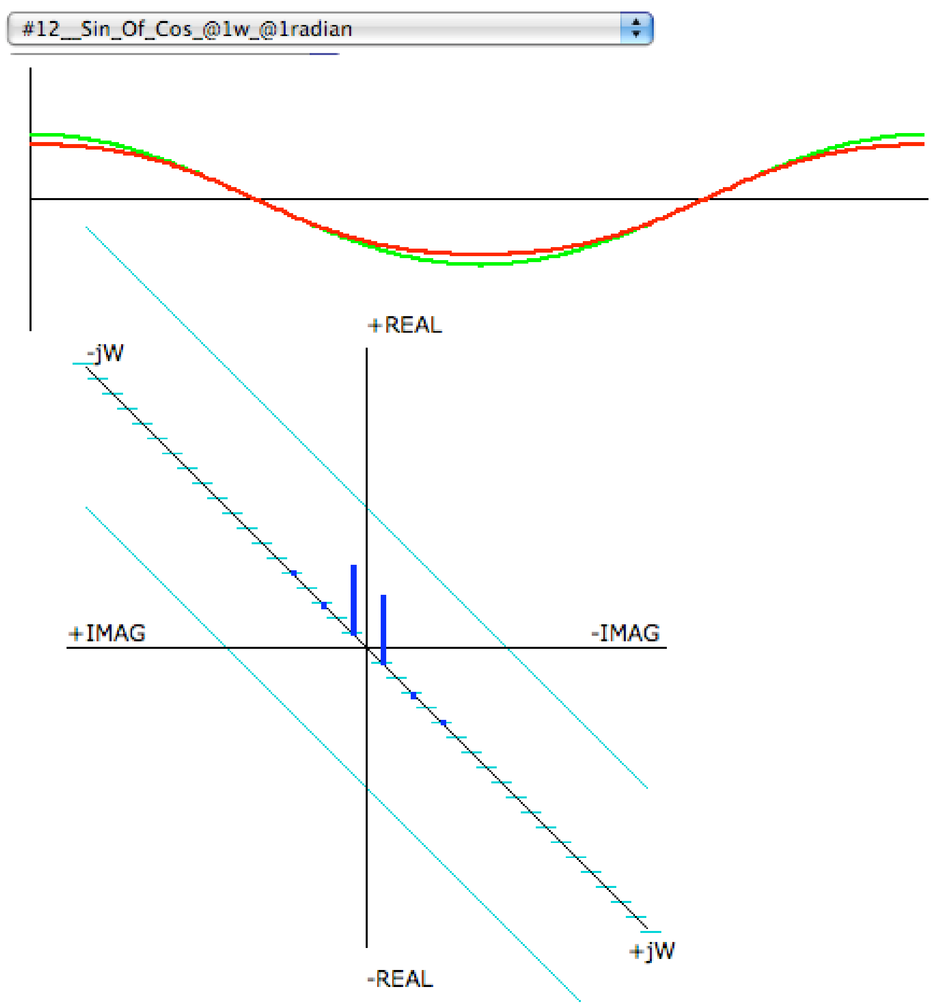
Over a +/- one radian swing the projection to the cosine plane will vary from one to about one half. But look at the actual fft values.

wt/wt_0	Real	Imaginary
0	0.7652	0.000
1	0.000	0.000
2	-0.1149	0.000
3	0.000	0.000
4	0.002477	0.000
5	0.000	0.000
6	0.000	0.000

β	J0	J1	J2	J3	J4	J5	J6	J7	J8
0	1								
0.25	0.98	0.12							
0.5	0.94	0.24	0.03						
1.0	0.77	0.44	0.11	0.02					
2.0	0.22	0.58	0.35	0.13	0.03				
3.0	-0.26	0.34	0.49	0.31	0.13	0.04	0.01		
4.0	-0.40	-0.07	0.36	0.43	0.28	0.13	0.05	0.02	
5.0	-0.18	-0.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02

Table 1. Bessel Functions of the First Kind Rounded to Two Decimal Places.

Notice how they fit some of the bessel values at Beta = 1.
Now do the sine plane projection



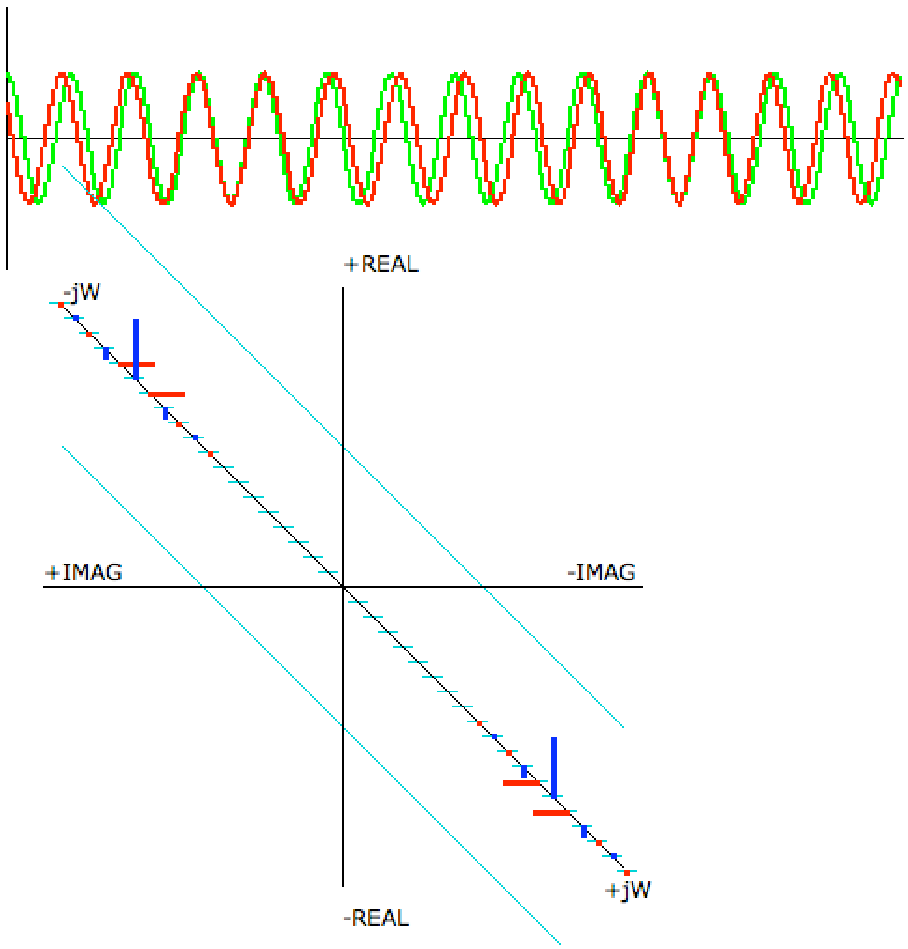
In the sine plane the signal is almost unity gain.
Look at its harmonics. Here are the rest of the bessel values.

wt/wt_0	Real	Imaginary
0	0.000	0.000

1	0.4401	0.000
2	0.000	0.000
3	-0.01956	0.000
4	0.000	0.000
5	0.0002498	0.000
6	0.000	0.000

So when the signal gets linearly phase modulated, the polar to rectangular translation will both tend to provide distorted AM and PM sidebands.

#9___Phase_Modulation___Cos_@14w_by_Cos_@1w_@1rad



The AM sidebands will be vertical and come form the projection to the cosine plane and the PM sidebands will come from the projection to the sine plane.

What about +/-5 radians of modulation?

#15__Cos_Of_Cos_@1w_@5radian

wt/wt_0	Real	Imaginary
0	-0.1776	0.000
1	0.000	0.000
2	-0.04657	0.000
3	0.000	0.000
4	0.3912	0.000
5	0.000	0.000
6	-0.1310	0.000
7	0.000	0.000
8	0.01841	0.000

9	0.000	0.000
10	-0.001468	0.000

#16__Sin_Of_Cos_@1w_@5radian

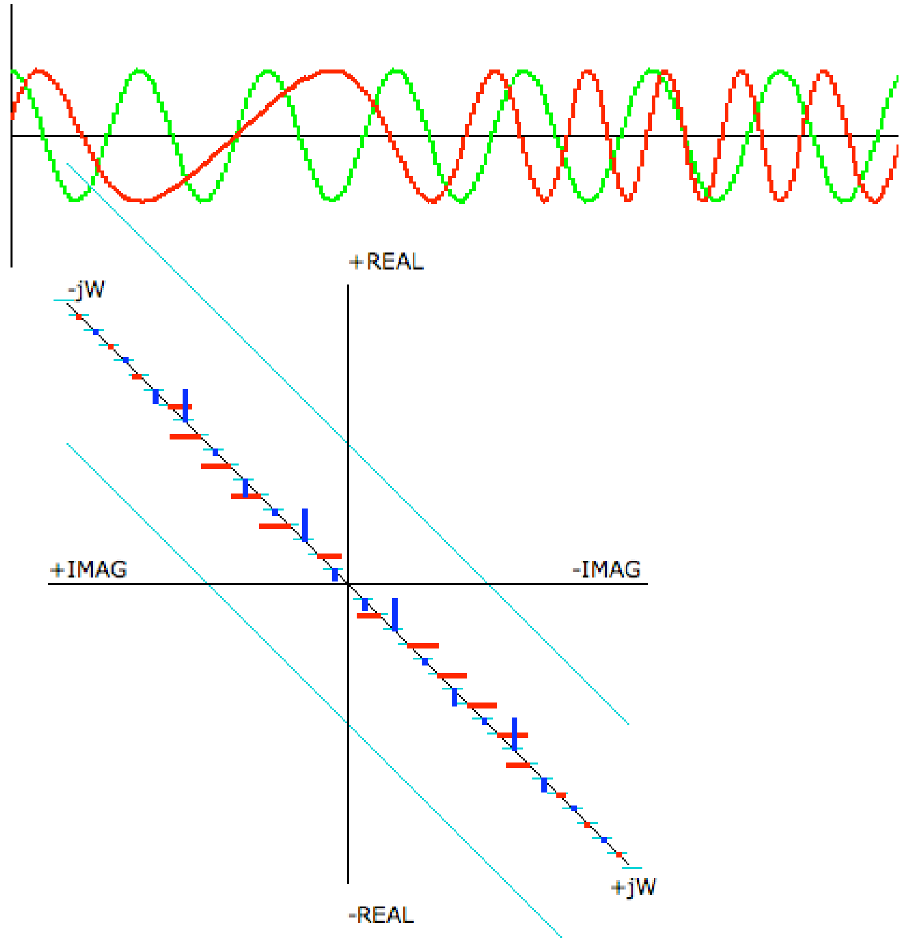
wt/wt_0	Real	Imaginary
0	0.000	0.000
1	-0.3276	0.000
2	0.000	0.000
3	-0.3648	0.000
4	0.000	0.000
5	0.2611	0.000
6	0.000	0.000
7	-0.05338	0.000
8	0.000	0.000
9	0.005520	0.000
10	0.000	0.000
11	-0.0003509	0.000
12	0.000	0.000

β	J0	J1	J2	J3	J4	J5	J6	J7	J8
0	1								
0.25	0.98	0.12							
0.5	0.94	0.24	0.03						
1.0	0.77	0.44	0.11	0.02					
2.0	0.22	0.58	0.35	0.13	0.03				
3.0	-0.26	0.34	0.49	0.31	0.13	0.04	0.01		
4.0	-0.40	-0.07	0.36	0.43	0.28	0.13	0.05	0.02	
5.0	-0.18	-0.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02

Table 1. Bessel Functions of the First Kind Rounded to Two Decimal Places.

Again all sidebands map to the the bessel function in magnitude. Now for +/- 5 radians of Frequency modulation, all sidebands can be accounted for. In the actual FM values will be a factor on 2 lower. (remember Euler identity format)

#17__FM/PM_Modulation___Cos_@7w_by_Cos_@1w_@5rad



wt/wt_0	Real	Imaginary
0	0.000	0.000
1	-0.05632	0.000
2	0.000	0.1278
3	0.1949	0.000
4	0.000	-0.1822
5	-0.02324	0.000
6	0.000	-0.1638
7	**-0.08880**	0.000
8	0.000	-0.1638
9	-0.02328	0.000
10	0.000	-0.1824
11	0.1956	0.000
12	0.000	0.1306
13	-0.06552	0.000
14	0.000	-0.02669
15	0.009203	0.000
16	0.000	0.002760
17	-0.0007339	0.000
18	0.000	-0.0001755
19	0.000	0.000