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## Energy harvesting from vibration using a piezoelectric membrane

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**Abstract.** In this paper we investigate the capability of harvesting the electric energy from mechanical vibrations in a dynamic environment through a unimorph piezoelectric membrane transducer. Due to the impedance matrices connecting the efforts and flows of the membrane, we have established the dynamic electric equivalent circuit of the transducer. In a first study and in order to validate theoretical results, we performed experiments with a vibrating machine moving a macroscopic 25 mm diameter piezoelectric membrane. A power of 1.8 mW was generated at the resonance frequency (2.58 kHz) across a 56 k $\Omega$  optimal resistor and for a 2 g acceleration.

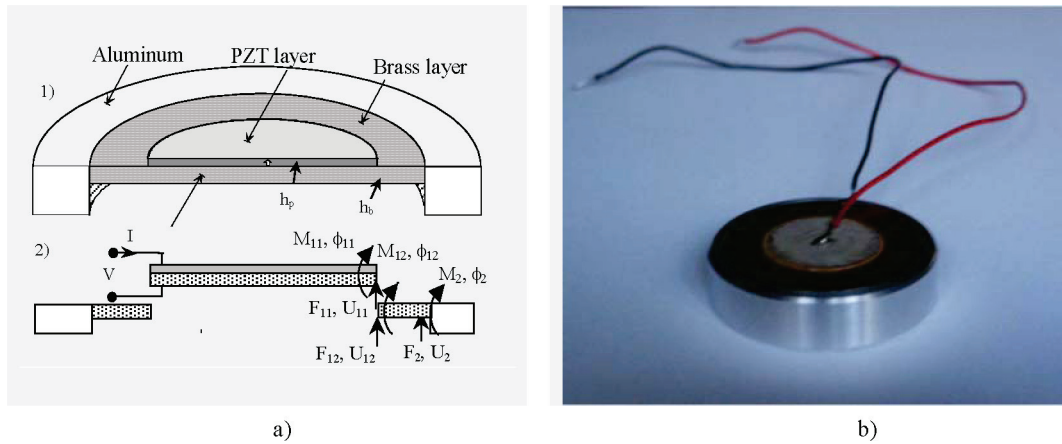
### 1. INTRODUCTION

In the last few years, the researches in the area of acquiring the energy surrounding a system and converting it into usable electrical energy have increased considerably [1]-[3]. These researches have been accelerated by the modern advances in wireless technology, low-power electronics and micromechanical systems. Due to the fact that these devices are portable, it becomes necessary to conceive portable power supply too. In particular, wireless sensors can be placed in very remote locations, for example a bridge whose cracks must be detected, and replacing the battery can become a very expensive task. If the energy of ambient vibration (i.e. bridge vibration) could be harvested, the battery could be reloaded and would not have to be replaced regularly. Thus, unlike cell phones or laptops, whose users can periodically recharge batteries, pervasive devices (sensors and actuators) must operate on their initial batteries. The highest reported energy densities for current Lithium-Ion battery technologies range around 0.9 kJ/cm<sup>3</sup>, which implies that for a low-power device operating at an average consumption of 1 mW and a 10-year lifespan, it needs a large 400 cm<sup>3</sup> battery. Thus, energy supply is a major drawback for system lifetime and harvesting energy from the deployment environments can help alleviate this.

Moreover, these sensors and actuators strive to meet application performance requirements using only environmentally available energy and can thus sustain themselves infinitely. In contrast to battery-operating systems, power management in energy harvesting systems differs fundamentally in that it is the available power that is limited and not the total energy. Also, power availability varies in time. Clearly, the system lifetime depends on how energy will be managed by the electronic circuits.

The use of piezoelectric materials to recover ambient vibrations surrounding a system is a method that is attracting a growing interest. Piezoelectric materials have a crystalline structure that enables them to transform mechanical strain energy into electrical energy, particularly for high frequency strains. This property makes them competitive compared to electromagnetic transducers.

In this study, we investigate the feasibility of harvesting energy from mechanical vibrations in dynamic environment using a diaphragm piezoelectric transducer. The transducer consists of an unimorph circular membrane (Lead Zirconate Titanate (PZT)/Brass) (Fig. 1-a). The mechanical vibrations cause an oscillating motion of the membrane. The resulting strain on the piezoelectric layer generates a low-frequency AC voltage signal along the electrode layer. It is converted into DC voltage when charging



**Figure 1.** a) Cross-section of the unimorph membrane transducer, b) picture of the transducer.

a storage capacitor through a rectifier. The piezoelectric membrane consists of multiple layers but two layers are used here: a bottom non-active layer (core) and an active layer of piezoelectric material attached to the upper side of the non-active layer. The core is typically a thick brass layer designed to increase the bending moment around, which moves the other layers. The generating layer can be made of any durable piezoelectric or electroactive material, though in this paper we use only PZT. The goal is to maximize the strain in the active layers.

The objective of the work described herein is to develop a piezoelectric generator that maximizes the power generated from a mechanical vibration. This paper initially presents the piezoelectric device, then an analytical model with impedance matrices of the piezoelectric membrane is introduced and the electric equivalent circuit is presented. Finally, the paper shows the experimental setup with some experimental results that validate our first prototype performances.

## 2. UNIMORPH MEMBRANE TRANSDUCER

The unimorph membrane transducer consists of two layers (PZT/brass). The total radius is  $R_1 = 20.5\text{mm}$ , the PZT radius is  $R_p = 12.5\text{mm}$ , the brass thickness is  $h_b = 400\mu\text{m}$  and the PZT thickness is  $h_p = 230\mu\text{m}$ . The brass layer is embedded over the whole circumference by epoxy adhesive. We can consider that embedding is perfect.

For a small amplitude of vibration a classical massive piezoelectric transducer cannot supply a sufficient electric energy amount because the deformation is very small. From this point of view, we have chosen to use a transducer working with flexion vibrations. In this case, a very small vibration gives a consequent deformation of the membrane which generates electric energy.

The selection of the piezoelectric material has been achieved regarding the value of the product  $\Pi = d \cdot g$  of the material. ( $d$  is piezoelectric field constant and  $g$  is the piezoelectric voltage constant). For a specified value of area and thickness, the electrical power depends on this product. A material with a high  $\Pi$  will generate a high power. Table I shows that material P-7 has the highest magnitude of  $\Pi$  because of its lowest dielectric constant rather than material P-7B.

The highest value of  $\Pi$  is obtained when the electromechanical coupling factor ( $k_{31}$ ) has the highest value. The input mechanical energy is converted to electrical energy, the ratio depending on the square of the electromechanical coupling factor. The remaining of energy is stored as mechanical energy which can be reduced by the choice of a lower dielectric constant.

**Table 1.** PZT material parameters “Murata”.

	P- 5E	P- 6F	P- 7	P- 7B
$\epsilon_r$	1510	1780	2100	4720
$d_{31}$	-131	-148	-207	-303
$g_{31}$	-10	-9	-11	-7
d.g	1310	1332	2277	2121

### 3. EQUIVALENT ELECTRIC CIRCUIT OF THE TRANSDUCER

The electrical behavior of a vibrating piezoelectric transducer can be modelled by an equivalent electric circuit. Starting from the electromechanical equations describing the system, we established the impedance matrices of the two parts of the circular membrane: the central part (PZT & brass) and the annular part (brass). Background and validation for the impedance matrix theory is contained in [3] and a brief summary is presented hereunder. Then, we connect them by considering the continuity of the strains and velocities at the mechanical junctions of the two elements.

Mechanical quantities, represented in figure 1-a, are: the bending moment noted  $M_{11}$ , the angular velocity  $\Phi_{11}$ , the shear force  $F_{11}$  and the linear velocity  $U_{11}$  at position  $r = R_1$ , and the same quantities ( $M_2, \Phi_2, F_2, U_2$ ) at the position  $r = R_2$ . Forces and velocities at the end of each element are indicated. For the central part, the linear relationship between the efforts and the flows is expressed by a 3x3 impedance matrix:

$$\begin{bmatrix} F_{11} \\ M_{11} \\ V \end{bmatrix} = \begin{bmatrix} Z_{11}^{Central} & Z_{12}^{Central} & 0 \\ & Z_{22}^{Central} & Z_{23}^{Central} \\ Sym & & Z_{33}^{Central} \end{bmatrix} \begin{bmatrix} U_{11} \\ \phi_{11} \\ I \end{bmatrix} \quad (1)$$

The elements of the impedance matrix are:

$$\begin{aligned} Z_{11}^{Central} &= \frac{4\pi R_1 K_u \lambda^3}{j\omega} \cdot \frac{J_1(\lambda R_1) I_1(\lambda R_1)}{J_0(\lambda R_1) I_1(\lambda R_1) + J_1(\lambda R_1) I_0(\lambda R_1)} \\ Z_{12}^{Central} &= \frac{-2\pi R_1 K_u \lambda^2}{j\omega} \cdot \frac{J_0(\lambda R_1) I_1(\lambda R_1) - J_1(\lambda R_1) I_0(\lambda R_1)}{J_0(\lambda R_1) I_1(\lambda R_1) + J_1(\lambda R_1) I_0(\lambda R_1)} \\ Z_{22}^{Central} &= \frac{2\pi K_u \lambda (1 - \sigma)}{j\omega} + \frac{4\pi K_u \lambda^2 R_1 J_0(\lambda R_1) I_0(\lambda R_1)}{j\omega (J_0(\lambda R_1) I_1(\lambda R_1) + J_1(\lambda R_1) I_0(\lambda R_1))} + \frac{(2\pi R_1 N)^2}{jC\omega} \\ Z_{23}^{Central} &= \frac{-2\pi N R_1}{jC\omega}; \quad Z_{33}^{Central} = \frac{1}{jC\omega} \end{aligned} \quad (2)$$

where  $J_0$  is the first form of Bessel function,  $I_0$  is the modified first form of Bessel function,  $K_u$  is the central part membrane bending stiffness,  $\lambda$  is the wavelength,  $N$  is a constant related to electromechanical conversion,  $\sigma$  is the Poisson's ratio and  $C$  is the input capacitance.

For the annular part, the linear relationship between the efforts and the flows is expressed by a 4x4 impedance matrix. It can be simplified into a 2x2 matrix because the velocities at the embedded

side are null ( $U_2=0$  and  $\Phi_2=0$ ). In this case, the linear relationship between the efforts and the flows is expressed by:

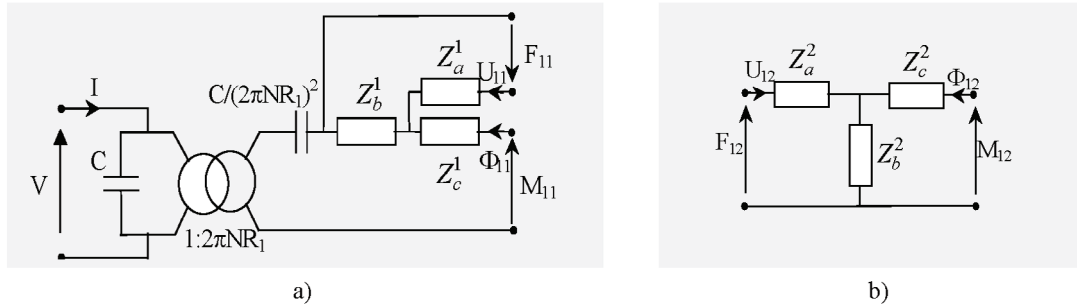
$$\begin{bmatrix} F_{12} \\ M_{12} \end{bmatrix} = \begin{bmatrix} Z_{11}^{Annular} & Z_{12}^{Annular} \\ Z_{21}^{Annular} & Z_{22}^{Annular} \end{bmatrix} \begin{bmatrix} U_{12} \\ \phi_{12} \end{bmatrix} \quad (3)$$

$Z^{Annular}$  is the mechanical impedances matrix of the annular membrane. This expression is not included on this paper, they can be found in [3].

From these matrices, we draw out an equivalent electric circuit by connecting the electrically equivalent impedances to the circuit according to the mechanical boundary conditions: the continuity of the mechanical quantities at the junction implies:  $M_{11} = -M_{12}$  and  $F_{11} = -F_{12}$ . There are represented in figure 2-b. The impedances of the electric circuit are:

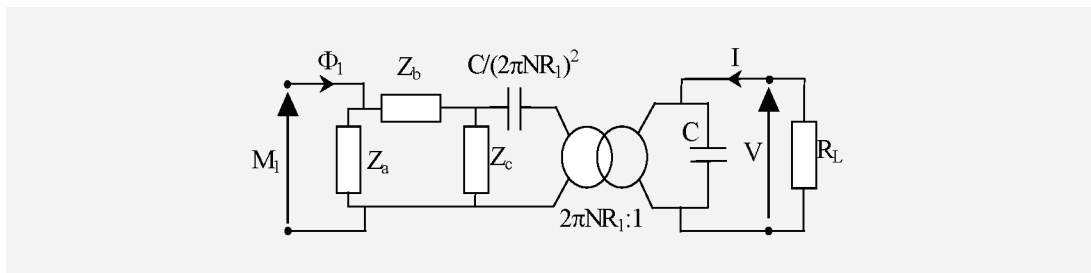
$$Z_a^1 = Z_{11}^{Central} - Z_{12}^{Central}; \quad Z_b^1 = Z_{12}^{Central}; \quad Z_c^1 = Z_{22}^{Central} - Z_{12}^{Central} \quad (4)$$

$$Z_a^2 = Z_{11}^{Annular} - Z_{12}^{Annular}; \quad Z_b^2 = Z_{12}^{Annular}; \quad Z_c^2 = Z_{22}^{Annular} - Z_{12}^{Annular} \quad (5)$$



**Figure 2.** Equivalent electric circuit of the transducer: a) central part, b) annular part.

The complete electric equivalent circuit of the unimorph membrane transducer is achieved by assembling the two previous simple circuits.



**Figure 3.** Equivalent electric circuit of the transducer.

The voltage generated by the piezoelectric element can now be expressed as a function of the bending moment  $M_1$  and the load resistance  $R_L$ :

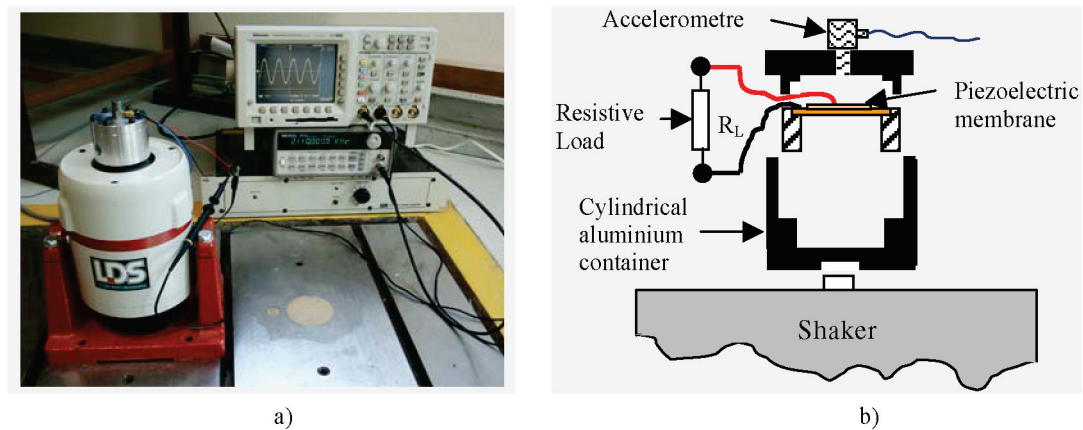
$$\frac{V}{M_1} = 1/2\pi NR_1 \left[ \frac{1 + j2R_L C \omega}{jR_L C \omega} \left( \frac{Z_b + Z_c}{Z_c} \right) + \left( \frac{1}{R_L} jC \omega \right) \frac{Z_b}{(2\pi NR_1)^2} \right] \quad (6)$$

In this model, the mechanical losses have not taken into account, however they can be introduced by a resistor  $R$  in series with the mechanical branch. The bending Moment  $M_1$  is proportional to the acceleration.

## 4. EXPERIMENTS

### 4.1 Experimental setup

Figure 4 shows pictures of the experimental setup. In this figure, the mechanical shaker is linked to a high power amplifier and connected to a function generator.



**Figure 4.** a) Experimental setup. b) Schematic view of the membrane's installation inside her support.

The membrane transducer and an accelerometer are rigidly fixed at the top of the shaker. In order to avoid any interference from the noise in surrounding environment, the shaker is placed on an isolated bench. The output voltage issued from the membrane transducer is monitored thanks to a Tektronix digital oscilloscope. In the first study, the AC voltage is directly applied to the variable resistor. In this case the harvested power is calculated by relation 7:

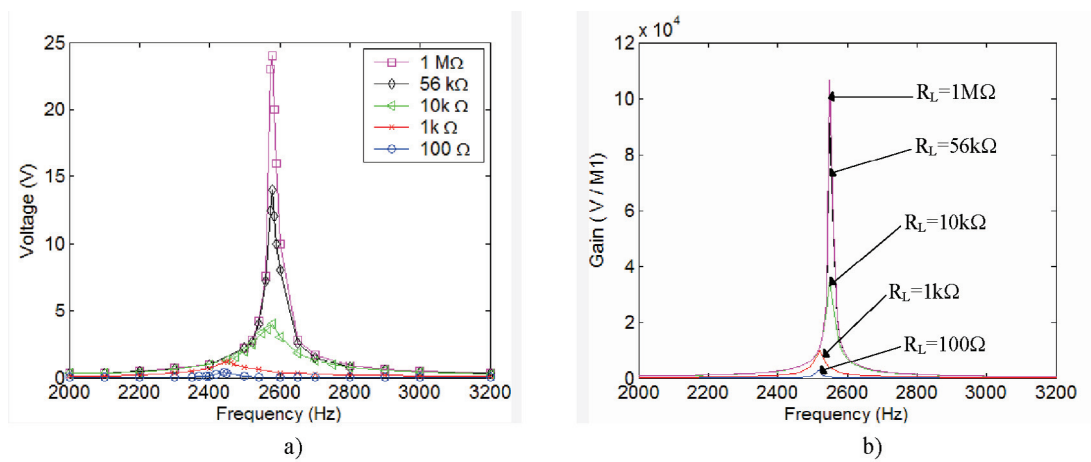
$$P = \frac{V_{Max}^2}{2R_L} \quad (7)$$

### 4.2 Experimental results

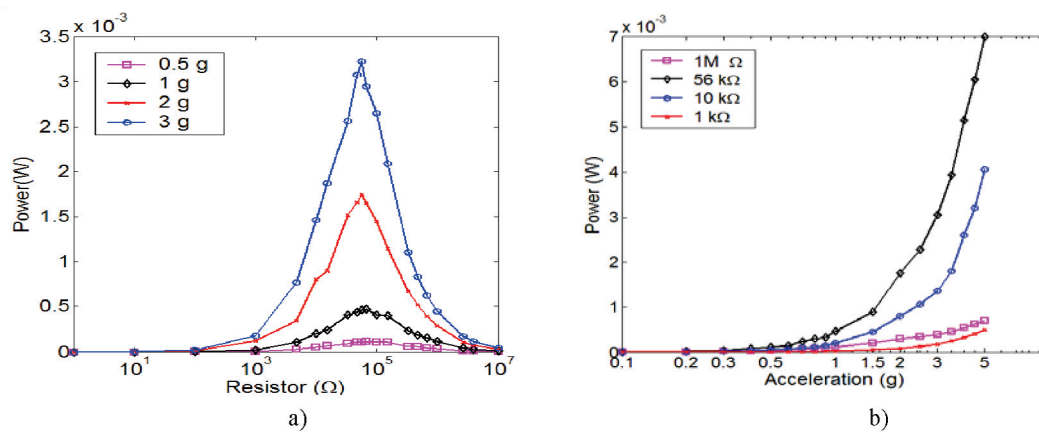
Figure 5a and 5b show respectively the experimental and the simulated results. Figure 5a shows the output voltage versus frequency for a 2 g acceleration and parametered by five values of load resistance. At the resonance frequency (2.58kHz) and when the generator is unloaded ( $R_L = 1M\Omega$ ) the maximum voltage obtained is: 24V. Figure 5b shows the corresponding calculated gain results from equation 6.

A good similarity between the experiment and simulation results can be noted which validates our electric equivalent circuit. The maximum amplitudes of the curves of figures 5-b are conditioned by losses; thus, these values are only indicative, they do not reflect the real material gain amplitude.

Figures 6-a shows the experimental output power versus the load resistance for four different acceleration values. We can see that the output voltage and power increase with acceleration. For a load resistance of 56k $\Omega$  the maximum power is observed. For a 2g acceleration an output power of 1.8mW is obtained. Figure 6-b shows the relation between output power and acceleration.



**Figure 5.** a) Experimental output voltage versus frequency (acceleration of 2g). b) Simulated gain  $V / M_1$ .



**Figure 6.** a) Power according the resistive load. b) Power versus acceleration.

The output voltage increases with the load. On the other hand, the maximum power value reaches a maximum of 1.8mW corresponding to a 56 kΩ load resistance. All above experimental results show that the power harvested can be improved when operating at the resonance frequency of the membrane, at high acceleration and especially with an optimal load resistance.

## 5. CONCLUSION

We have realized and modelled an energy harvesting generator based on the use of a piezoelectric unimorph membrane structure operating in flexion mode. A maximum power of 1.8mW was generated under an acceleration of 2g and a load resistance of 56 kΩ. An analytical modelling of this generator was carried out and an equivalent electromechanical circuit has been proposed. The analytical results were compared successfully to experiments. Future works will consist of a more precise study of the power available at the output of the generator and will concern the design of the electronic circuit aimed to increase the power harvested by the piezoelectric device. The first experimentation with this electrical converter will apply the SSHI technique “Synchronized Switching Harvesting with Inductor” [4]. This

technique gives remarkable results: the generated power can be significantly multiply at resonance (literature relates 4 to 5 times), leading to sufficient power to supply a large range of low consumption sensors.

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## FURTHER READING

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[3941402\\_Electromagnetic\\_shock\\_absorber](#)  
[4032829\\_Road\\_shock\\_energy\\_converter](#)  
[5347186\\_Linear\\_motion\\_electric\\_power\\_generator](#)  
[5818132\\_Linear\\_motion\\_electric\\_power\\_generator](#)  
[6952060\\_Electromagnetic\\_linear\\_generator](#)  
[7357229\\_Electromagnetic\\_shock\\_absorber](#)  
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[electromagnetic energy harvester for vehicle suspensions](#)  
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[Energy harvesting from vibration](#)  
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